Physics 780 – General Relativity

Homework Set W

54. In class I assumed the identity $\int T^{0i}\left(x\right)x^j d^3\mathbf{x} = \int T^{0j}\left(x\right)x^i d^3\mathbf{x}$. This isn't quite true, but it is close to true.
   (a) Call the difference between these two expressions J^{ij}. Write an expression for the time derivative of J^{ij}.
   (b) Use the identity $\partial_0 T^{0\alpha} = -\partial_k T^{k\alpha}$ to rewrite the integrals as space derivatives.
   (c) Integrate these expression by parts. Since we are going to assume T vanishes at sufficient distances, the surface terms vanish. Simplify the remaining derivatives by using $\partial_k x^\ell = \delta_k^\ell$, and do the sum over k.
   (d) Show that J^{ij} is constant. Since we are focusing on the portion of the integrals that oscillate, this means that any oscillating component satisfies $\int T^{0i}\left(x\right)x^j d^3\mathbf{x} = \int T^{0j}\left(x\right)x^i d^3\mathbf{x}$.

55. We had some angular integrals that needed to be done, of the form $\int d\Omega$, $\int k_i k_j d\Omega$ and $\int k_i k_j k_\ell k_m d\Omega$, where $\int d\Omega \equiv \int_0^{2\pi} d\phi \int_0^\pi \sin\theta\, d\theta$.
   (a) Find $\int d\Omega$. This part of the problem is completely different from the remaining parts.
   (b) To find $\int k_i k_j d\Omega$, first note that since all directions are created equal, it must be some sort of invariant tensor. The only tensors in 2D that are invariant are δ_{ij} and $\tilde{\varepsilon}_{ijk}$ and combinations of them. Argue that the result must be proportional to δ_{ij}. Call the constant of proportionality A.
   (c) Multiply the integral in part (b) by δ_{ij} summing over i and j, and using the identity $\mathbf{k}^2 = \omega^2$ to simplify. Determine A.
   (d) To find $\int k_i k_j k_\ell k_m d\Omega$, first note that since all directions are created equal, it must be some sort of invariant tensor. Argue that the result must be proportional to $\delta_{ij}\delta_{\ell m} + \delta_{i\ell}\delta_{jm} + \delta_{im}\delta_{j\ell}$. Call the constant of proportionality B.
   (e) Do something similar to what you did in part (c) and use the identity $\mathbf{k}^2 = \omega^2$ to simplify and determine B.