

Static Spherically Symmetric Spacetimes

The metric: $ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

Non-zero Christoffel symbols:

$$\Gamma_{tr}^t = \Gamma_{rt}^t = \frac{f'}{2f}, \quad \Gamma_{tt}^r = \frac{f'}{2h}, \quad \Gamma_{rr}^r = -\frac{h'}{2h}, \quad \Gamma_{r\theta}^\theta = \Gamma_{\theta r}^\theta = \Gamma_{r\phi}^\phi = \Gamma_{\phi r}^\phi = \frac{1}{r},$$

$$\Gamma_{\theta\theta}^r = -\frac{r}{h}, \quad \Gamma_{\phi\phi}^r = -\frac{r \sin^2\theta}{h}, \quad \Gamma_{\theta\phi}^\phi = \Gamma_{\phi\theta}^\phi = \cot\theta, \quad \Gamma_{\phi\phi}^\theta = -\sin\theta \cos\theta.$$

The Ricci Tensor:

$$R_{tt} = \frac{f''}{2h} - \frac{f'^2}{4fh} - \frac{fh'}{4h^2} + \frac{f'}{rh}, \quad R_{rr} = -\frac{f''}{2f} + \frac{f'^2}{4f^2} + \frac{fh'}{4fh} + \frac{h'}{hr}, \quad R_{\theta\theta} = -\frac{f'r}{2fh} + \frac{rh'}{2h^2} + 1 - \frac{1}{h},$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta}.$$

Ricci scalar:

$$R = -\frac{f''}{fh} + \frac{f'^2}{2f^2h} + \frac{fh'}{2fh^2} - \frac{2f'}{fhr} + \frac{2h'}{h^2r} + \frac{2}{r^2} - \frac{2}{r^2h}$$