Physics 745 - Group Theory

Homework Set 21

Due Friday, March 27

1. In class (or the notes), I gave explicit instructions for how to find the irreducible representations $T_a^{(j)}$. To demonstrate that you understand this, write explicitly $T_3^{(2)}$, $T_\pm^{(2)}$, $T_1^{(2)}$, and $T_2^{(2)}$ for the $j = 2$ irrep. Check that it is correct by computing

$$\mathbf{T}^2 = T_1^2 + T_2^2 + T_3^2$$

and show that it has the correct value.

2. This problem has to do with breaking down an unknown representation of $SO(3)$ into irreps.
   (a) Using the highest weight decomposition described in the notes, work out the decomposition of the defining generators of $SO(3)$, given in equation (2.5):

$$T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

   (b) A certain representation of $SO(3)$ has generators given by

$$T_1 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}, \quad T_2 = \frac{i}{2}\begin{pmatrix} 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}, \quad T_3 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

   How does this break down into irreps?