Physics 745 - Group Theory

Solution Set 17

1. [10] For the group *O*, with character table given on page 329 of Tinkham, work out the breakdown (compatibility) for the tensor product $\Gamma^{A\otimes B}$ for every pair of irreps (15 in all).

$\mathcal{O}$	E	$8C_3$	$3C_4^2$	$6C_2$	$6C_4$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
T_1	3	0	-1	-1	1
T_2	3	0	-1	1	-1
$A_2\otimes A_2$	1	1	1	1	1
$A_2\otimes E$	2	-1	2	0	0
$A_2\otimes T_1$	3	0	-1	1	-1
$A_2\otimes T_2$	3	0	-1	-1	1
$E\otimes E$	4	1	4	0	0
$E\otimes T_1$	6	0	-2	0	0
$E\otimes T_2$	6	0	-2	0	0
$T_1\otimes T_1$	9	0	1	1	1
$T_1\otimes T_2$	9	0	1	-1	-1
$T_2\otimes T_2$	9	0	1	1	1

For A_1, the characters of $A_1\otimes\Gamma$ are obviously the same as Γ, so $A_1\otimes\Gamma=\Gamma$. This leaves ten other combinations, and the corresponding characters are listed in the table at right. The first four, by inspection, simply give irreps again. For $E\otimes E$, it is not hard to see that it contains E, and then what's left over is A_1 and A_2. For $E\otimes T_1=E\otimes T_2$, it is easy to see that it is just $T_1\oplus T_2$. For $T_1\otimes T_1=T_2\otimes T_2$, we can use decomposition rules (if necessary) to see that they contain one copy each of T_1, T_2 each, and E. What's left over is then A_1. For $T_1\otimes T_2$, again you have one copy each of T_1, T_2 and E, and what's left over is A_2. In summary, we find

$$A_1\otimes A_1=A_1,\qquad A_2\otimes A_2=A_1,\qquad E\otimes T_1=T_1\oplus T_2,$$
$$A_1\otimes A_2=A_2,\qquad A_2\otimes E=E,\qquad E\otimes T_2=T_1\oplus T_2,$$
$$A_1\otimes E=E,\qquad A_2\otimes T_1=T_2,\qquad T_1\otimes T_1=T_1\oplus T_2\oplus E\oplus A_1,$$
$$A_1\otimes T_1=T_1,\qquad A_2\otimes T_2=T_1,\qquad T_1\otimes T_2=T_1\oplus T_2\oplus E\oplus A_2,$$
$$A_1\otimes T_2=T_2,\qquad E\otimes E=E\oplus A_1\oplus A_2,\qquad T_2\otimes T_2=T_1\oplus T_2\oplus E\oplus A_1.$$