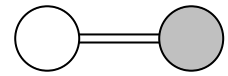
Physics 745 - Group Theory

Solution Set 33

1. The group SO(5) has the Dynkin diagram sketched at right. The shorter root can be chosen to be $\mathbf{s} = (0,1)$.



(a) What is the length of the longer root r? Give the coordinates of r.

The longer root will be $\sqrt{2}$ longer than **s**, and it must be at 135° angle compared to it. Therefore, the longer root will be of length $\sqrt{2}$ and have coordinates

$$\mathbf{r} = (1,-1).$$

(b) Use the rules described in class to determine for what positive integers *n* the quantities $\mathbf{r}+n\mathbf{s}$ and $\mathbf{s}+n\mathbf{r}$ are roots. Write them all out in coordinates.

We can only add simple roots, we can't subtract them. Since $2\mathbf{r}\cdot\mathbf{s}/\mathbf{r}^2 = -1$, we can only add **r** to **s** once. Since $2\mathbf{r}\cdot\mathbf{s}/\mathbf{s}^2 = -2$, we can add **s** to **r** twice. This yields two new roots, namely,

$$\mathbf{r}+\mathbf{s} = \mathbf{s}+\mathbf{r} = (1,0) \quad \text{and} \quad \mathbf{r}+2\mathbf{s} = (1,1)$$

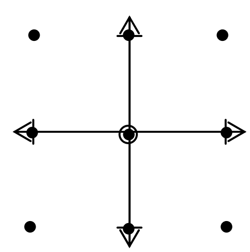
(c) Prove or disprove: More roots can be found by adding r or s to the positive roots we have already found.

We know we can add neither **r** nor **s** to **r** + **s**. We know we can't add **s** to **r** + 2**s**. Can we add **r** to it?

$$2(\mathbf{r}+2\mathbf{s})\cdot\mathbf{r}/\mathbf{r}^2 = 2(1,1)\cdot(1,-1)/2 = 0.$$

No, we can't add any more.

(d) You have found all the positive roots. Find all the negative roots. Find all the zero roots. Make a root diagram. It should be a nice, symmetric pattern.



The negative roots are the negatives of the positive roots, or

$$\{(0,-1),(-1,1),(-1,0),(-1,-1)\}$$

There are also two zero roots. These are all plotted in the root diagram above.