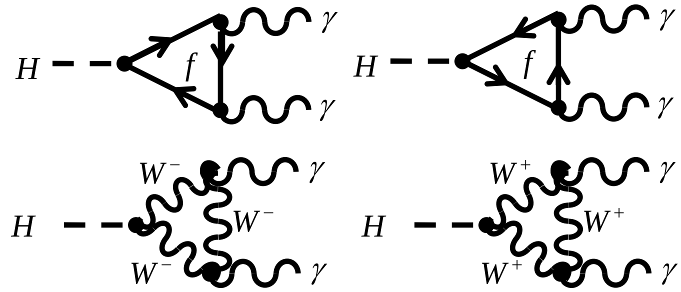
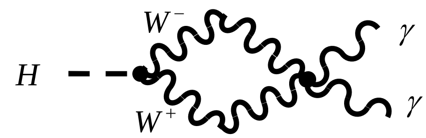
## Solutions to Problems 11b

**4. The Higgs boson was actually discovered by the decay $H \to \gamma\gamma$. This process is impossible by tree diagrams. Draw some loop diagrams, including at least one with no fermion loop, that would contribute to this process. Speculate which diagram(s) you think might be most important.**




At one loop level, there are five diagrams, which I have sketched. The fermion *f* can be any fermion. The two *W*-diagrams look almost identical, but technically they are different, since one represents a $W^+$ going around clockwise, and the other has a $W^-$ going around clockwise. For the fermion loop, the large top quark coupling makes the top quark loop important, but the large mass also decreases the rate. The other, lighter fermions don't contribute much because of their much smaller couplings. The *W*-loop diagrams are certainly important as well, and I think actually are the dominant contribution.




**5. It is sometimes said that a non-zero $\delta$ is what causes CP violation, because it makes the CKM matrix eq. (10.61) complex. It isn't quite this simple.**

**(a) Argue that the CKM matrix is real also if $\delta = \pi$ or if $s_{13} = 0$.**

The CKM matrix is given by

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

Noting that $e^{i\pi} = -1$, it is clear that $\delta = \pi$ also makes the matrix real. It is also easy to see that $e^{i\delta}$ is always multiplied by $s_{13}$, so if $s_{13} = 0$ the matrix is manifestly real.

**(b) The Standard Model is unchanged if the CKM matrix is multiplied on the left or right by just diagonal phases, for example, we can change it to $V \to U^*VU$, where**

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}, \quad U^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix}.$$

**show that if $s_{23} = 0$, this will make the CKM matrix *V* change to a real matrix.**

If $s_{23} = 0$ (which incidentally implies $c_{23} = 1$), the CKM matrix simplifies to

$$V=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} & c_{12} & 0 \\ -c_{12}s_{13}e^{i\delta} & -s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

We therefore have

$$U^*VU=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix}\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} & c_{12} & 0 \\ -c_{12}s_{13}e^{i\delta} & -s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} & c_{12} & 0 \\ -c_{12}s_{13} & -s_{12}s_{13} & c_{23}c_{13} \end{pmatrix},$$

which is manifestly real.

**(c) Find a different diagonal matrix *U* such that if $s_{12}=0$, then $V \to U^*VU$ will make the CKM matrix *V* come out to a real matrix.**

Setting $s_{12}=0$ (which also implies $c_{12}=1$), the CKM matrix simplifies to

$$V=\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ -s_{23}s_{13}e^{i\delta} & c_{23} & s_{23}c_{13} \\ -c_{23}s_{13}e^{i\delta} & -s_{23} & c_{23}c_{13} \end{pmatrix}.$$

If we then set

$$U=\begin{pmatrix} e^{-i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U^*=\begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

then we have

$$U^*VU=\begin{pmatrix} e^{i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ -s_{23}s_{13}e^{i\delta} & c_{23} & s_{23}c_{13} \\ -c_{23}s_{13}e^{i\delta} & -s_{23} & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} e^{-i\delta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}=\begin{pmatrix} c_{13} & 0 & s_{13} \\ -s_{23}s_{13} & c_{23} & s_{23}c_{13} \\ -c_{23}s_{13} & -s_{23} & c_{23}c_{13} \end{pmatrix}.$$

Again, this is manifestly real.

**7. Are we sure that the neutrino is *exactly* neutral? Are we sure that the proton *exactly* cancels the electron's charge? Let's define the Higgs field to have $Y=+\frac{1}{2}$ (this just sets the overall scale), but then change the various charges of the fermions to**

left-fermions: $3\left[\left(3,2,\frac{1}{6}+\delta\right)\oplus\left(\overline{3},1,-\frac{2}{3}-\delta\right)\oplus\left(\overline{3},1,+\frac{1}{3}-\delta\right)\oplus\left(1,2,-\frac{1}{2}+\varepsilon\right)\oplus\left(1,1,+1-\varepsilon\right)\right].$

**This form is in fact required to make the Higgs couplings to work out.**

**(a) Using $Q=T_3+Y$, find the charge of the electron, the neutrino, the up quark, the down quark, the proton, and the hydrogen atom.**

For the left-handed quark doublet, we get $Q=T_3+Y=\pm\frac{1}{2}+\frac{1}{6}+\delta$, which gives an up quark mass of $Q_u=\frac{2}{3}+\delta$ and a down quark charge of $Q_d=-\frac{1}{3}+\delta$. For the left-handed lepton doublet, we have $Q=T_3+Y=\pm\frac{1}{2}-\frac{1}{2}+\varepsilon$, which yields charge $Q_\ell=-1+\varepsilon$ and neutrino $Q_\nu=\varepsilon$. We can also work them out for the left-handed anti particles, which all have $T_3$ = 0, so the charge is just the hypercharge, $Q_{\bar{u}}=-\frac{2}{3}-\delta$, $Q_{\bar{d}}=\frac{1}{3}-\delta$, and $Q_{\bar{\ell}}=+1-\varepsilon$. Not surprisingly, the charges worked out equal and opposite, and therefore the particles have charge

$$Q_u=\frac{2}{3}+\delta,\quad Q_d=-\frac{1}{3}+\delta,\quad Q_e=-1+\varepsilon,\quad Q_\nu=\varepsilon.$$

The proton has two up quarks and a down quark. The hydrogen atom has a proton and an electron. Therefore

$$Q_p=1+3\delta,\quad Q_H=3\delta+\varepsilon\ .$$

**(b) Using some useful combination of the three anomaly conditions in problem 5, show that, in fact, $\delta=\varepsilon=0$.**

The simplest ones to check is

$$\begin{aligned}0=\sum_f Y_f &=18\left(\tfrac{1}{6}+\delta\right)+9\left(-\tfrac{2}{3}-\delta\right)+9\left(\tfrac{1}{3}-\delta\right)+6\left(-\tfrac{1}{2}+\varepsilon\right)+3\left(1-\varepsilon\right)\\ &=3+18\delta-6-9\delta+3-9\delta-3+6\varepsilon+3-3\varepsilon=3\varepsilon.\end{aligned}$$

So $\varepsilon=0$. We next note that $T_3^2=\frac{1}{4}$ for any member of a doublet, and $T_3^2=0$ for any member of a singlet, so

$$0=\sum_f T_3^2Y=18\left(\tfrac{1}{4}\right)\left(\tfrac{1}{6}+\delta\right)+6\left(\tfrac{1}{4}\right)\left(-\tfrac{1}{2}+\varepsilon\right)=\tfrac{3}{4}+\tfrac{9}{2}\delta-\tfrac{3}{4}+\tfrac{3}{2}\varepsilon=\tfrac{9}{2}\delta+\tfrac{3}{2}\varepsilon.$$

Since we already established that $\varepsilon=0$, this implies $\delta=0$.

Incidentally, if you add (the anti-particles of) three right-handed neutrinos $3\left[\left(1,1,-\varepsilon\right)\right]$, then the first equation will become 0 = 0 and the second will remain $3\delta+\varepsilon=0$. This means that the neutrino (and neutron) *could* have a tiny charge, but the hydrogen atom will still be neutral.