Solutions to Problems 6c

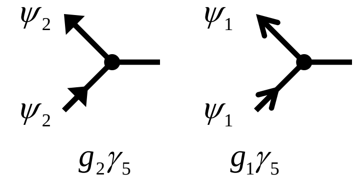
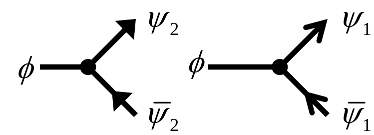


Figure 6-10: The couplings for problems 10 through 13.

10. Suppose we have two fermions ψ_1 and ψ_2 with masses m_1 and m_2, each of which has pseudoscalar couplings to the ϕ of mass *M*, with strength *g*₁ and *g*₂, as sketched in Fig. 6-10. What is the total decay rate of the ϕ?

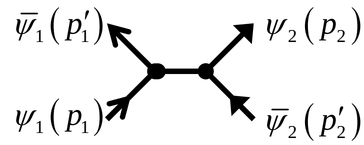
There are two processes involved, with the corresponding Feynman diagrams as sketched at right. These processes lead to different final states, and hence there is no interference. We simply calculate each of them separately. Since each of these processes is identical to the process calculated in section 6D, we can simply take over the result, eq. (6.18):



$$\Gamma\left(\phi \to \psi_1\bar{\psi}_1\right) = \frac{g_1^2}{8\pi}\sqrt{M^2 - 4m_1^2}\ , \quad \Gamma\left(\phi \to \psi_2\bar{\psi}_2\right) = \frac{g_2^2}{8\pi}\sqrt{M^2 - 4m_2^2}\ ,$$

$$\Gamma_{tot} = \Gamma\left(\phi \to \psi_1\bar{\psi}_1\right) + \Gamma\left(\phi \to \psi_2\bar{\psi}_2\right) = \frac{g_1^2}{8\pi}\sqrt{M^2 - 4m_1^2} + \frac{g_2^2}{8\pi}\sqrt{M^2 - 4m_2^2}\ .$$

11. Find the cross-section for $\psi_1\bar{\psi}_1 \to \psi_2\bar{\psi}_2$ in the theory of Fig. 6-10. Assume we are not near the ϕ resonance. Don't make any other approximations or assumptions.



There is only one Feynman diagram, sketched at right. The intermediate momentum is $p_1 + p_1'$. The Feynman amplitude for this process is therefore

$$i\mathcal{M} = \left(\bar{u}_2 g_2 \gamma_5 v_2'\right)\left(\bar{v}_1' g_2 \gamma_5 u_1\right)\frac{i}{\left(p_1 + p_1'\right)^2 - M^2} = \frac{ig_1 g_2\left(\bar{u}_2\gamma_5 v_2'\right)\left(\bar{v}_1'\gamma_5 u_1\right)}{s - M^2},$$

where *s* is the usual square of the center of mass energy. We now sum over final spins and average over initial, so we have

$$\frac{1}{4}\sum_{\text{spins}}\left|i\mathcal{M}\right|^2 = \frac{g_1^2 g_2^2}{4}\sum_{\text{spins}}\frac{\left(\bar{u}_2\gamma_5 v_2'\right)\left(-\bar{v}_2'\gamma_5 u_2\right)\left(\bar{v}_1'\gamma_5 u_1\right)\left(-\bar{u}_1\gamma_5 v_1'\right)}{\left(s - M^2\right)^2}$$

$$= \mathrm{Tr}\left[\left(\not{p}_2 + m_2\right)\gamma_5\left(\not{p}_2' - m_2\right)\gamma_5\right]\mathrm{Tr}\left[\left(\not{p}_1' - m_1\right)\gamma_5\left(\not{p}_1 + m_1\right)\gamma_5\right]\left(s - M^2\right)^{-2}$$

$$= \tfrac{1}{4} g_1^2 g_2^2 \mathrm{Tr}\left[\left(\not{p}_2 + m_2\right)\left(\not{p}_2' + m_2\right)\right]\mathrm{Tr}\left[\left(\not{p}_1' - m_1\right)\left(\not{p}_1 - m_1\right)\right]\left(s - M^2\right)^{-2}$$

$$= \tfrac{1}{4} g_1^2 g_2^2 \mathrm{Tr}\left(\not{p}_2\not{p}_2' + m_2^2\right)\mathrm{Tr}\left(\not{p}_1'\not{p}_1 + m_1^2\right)\left(s - M^2\right)^{-2}$$

$$= 4g_1^2 g_2^2\left(p_2 \cdot p_2' + m_2^2\right)\left(p_1 \cdot p_1' + m_1^2\right)\left(s - M^2\right)^{-2}.$$

The dot products are most easily worked out by noting that

$$s = \left(p_1 + p_1'\right)^2 = p_1^2 + p_1'^2 + 2p_1 \cdot p_1' = 2m_1^2 + 2p_1 \cdot p_1',$$

$$s = \left(p_2 + p_2'\right)^2 = p_2^2 + p_2'^2 + 2p_2 \cdot p_2' = 2m_2^2 + 2p_2 \cdot p_2'.$$

Substituting these in, we have

$$\frac{1}{4}\sum_{\text{spins}}\left|i\mathcal{M}\right|^2 = \frac{g_1^2 g_2^2 s^2}{\left(s - M^2\right)^2}.$$

We then compute the cross-section in the usual way.

$$\sigma = \frac{D}{8Ep_1} = \frac{1}{8Ep_1}\frac{p_2}{16\pi^2(2E)}\int\frac{1}{4}\sum_{\text{spins}}\left|i\mathcal{M}\right|^2 d\Omega = \frac{4\pi p_2}{256\pi^2 E^2 p_1}\frac{g_1^2 g_2^2 s^2}{\left(s - M^2\right)^2} = \frac{g_1^2 g_2^2 s p_2}{16\pi p_1\left(s - M^2\right)^2}$$

$$= \frac{g_1^2 g_2^2 s}{16\pi\left(s - M^2\right)^2}\sqrt{\frac{s - 4m_2^2}{s - 4m_1^2}}.$$