Midterm Information

Equations you should memorize:

$$\gamma_X = L\sigma_X \,, \tag{1.7}$$

$$N_X = \sigma_X \int L\,dt \,, \tag{1.9}$$

$$N_X(t) = N_{X0} \exp(-\Gamma_{\text{tot}} t) \,. \tag{1.12}$$

$$BR(X \to Y) \equiv \frac{\Gamma(X \to Y)}{\Gamma_{\text{tot}}(X)} \,. \tag{1.15}$$

$$c = \hbar = \varepsilon_0 = \mu_0 = 1 \,. \tag{1.18}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}} = \frac{L_0}{L} = \frac{t}{\tau} = \frac{E}{m} \,, \tag{2.8}$$

$$u \cdot v \equiv g_{\alpha\beta} u^\alpha v^\beta = u^0 v^0 - \mathbf{u} \cdot \mathbf{v} \,. \tag{2.11}$$

$$m^2 \equiv p \cdot p = E^2 - \mathbf{p}^2 \,. \tag{2.33}$$

$$\mathbf{v} = \frac{\mathbf{p}}{E} \,. \tag{2.34}$$

$$s \equiv (p_1 + p_2)^2 = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 \,. \tag{2.35}$$

$$p^\mu = (E, \mathbf{p}) = (E, p\sin\theta\cos\phi, p\sin\theta\sin\phi, p\cos\theta) \,.$$

$$\left(\bar{\Psi}_A \Gamma_1 \Gamma_2 \cdots \Gamma_n \Psi_B\right)^* = \bar{\Psi}_B \bar{\Gamma}_n \cdots \bar{\Gamma}_2 \bar{\Gamma}_1 \Psi_A \,. \tag{3.43}$$

$$\not{p} \equiv p_\mu \gamma^\mu \,. \tag{3.47}$$

$$\left| t_1, p_1, s_1 ; t_2, p_2, s_2 \right\rangle = \begin{cases} -\left| t_2, p_2, s_2 ; t_1, p_1, s_1 \right\rangle & \text{if two fermions ,} \\ \left| t_2, p_2, s_2 ; t_1, p_1, s_1 \right\rangle & \text{otherwise .} \end{cases} \tag{4.5}$$

Feynman Diagram things you should know:

- How to draw Feynman diagrams
- How to get the amplitudes from them:
  - Propagator for scalar: $i/(p^2 - m^2)$
  - Propagator for fermion: $i(\not{p} + m)/(p^2 - m^2)$
  - Follow fermion lines backwards from head to tail
  - External fermion rules will be given to you diagrammatically on info sheet
  - Vertex rules will be given to you diagrammatically as needed
  - Subtract diagrams with swapped fermion lines, otherwise add
- How to square them
  - Sum on final spins – average over initial spins
  - Turn them into traces for fermions
- How to get differential/total cross-sections and decay rates
  - Factor of $1/n!$ for identical final particles in total (not differential)

Other things you should understand:

- A little about how particle colliders work
- What cross-section and decay rate actually mean
- What parity and time reversal are
- Manifest Lorentz covariance
- Conservation of four-momentum, and how to use it
- How to determine if a basic matrix element is renormalizable
- How the Dirac equation predicts the existence of anti-particles
- What resonances mean, and how you can use them to measure Γ.

Useful Formulas and Identities – these will be provided

Units and Constants

$$1\text{ eV} = 1.602\times10^{-19}\text{ J}$$
$$6.582\times10^{-16}\text{ s}\cdot\text{eV} = 1$$
$$1\text{ s} = 3\times10^{8}\text{ m}$$
$$1\text{ kg} = 5.6\times10^{26}\text{ GeV}$$
$$197\text{ MeV}\cdot\text{fm} = 1$$
$$1\text{b} = 100\text{ fm}^2$$
$$m_e = 0.51100\text{ MeV}$$
$$m_p = 938.27\text{ MeV}$$

Metric Prefixes

T	10^{12}
G	10^{9}
M	10^{6}
k	10^{3}
m	10^{-3}
μ	10^{-6}
n	10^{-9}
p	10^{-12}
f	10^{-15}

Dirac Matrices (chiral representation)

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$
$$\boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}$$
$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Dirac Properties

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$
$$\{\gamma^\mu, \gamma_5\} = 0$$
$$\gamma_5\gamma_5 = 1$$
$$\bar{\Gamma} \equiv \gamma^0\Gamma^\dagger\gamma^0$$
$$\bar{\gamma}^\mu \equiv \gamma^\mu$$
$$\bar{\gamma}_5 = -\gamma_5$$

Dirac Trace Identities

$$\mathrm{Tr}\left(\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_{2N+1}}\right) = \mathrm{Tr}\left(\gamma_5\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_{2N+1}}\right) = 0$$
$$\mathrm{Tr}(1) = 4 \qquad \mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right) = 4g^{\mu\nu}$$
$$\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\right) = 4\left(g^{\mu\nu}g^{\alpha\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\alpha}g^{\nu\beta}\right)$$
$$\mathrm{Tr}(\gamma_5) = \mathrm{Tr}\left(\gamma_5\gamma^\alpha\gamma^\beta\right) = 0$$
$$\mathrm{Tr}\left(\gamma_5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\right) = -4i\varepsilon^{\mu\nu\alpha\beta}$$

Luminosity

$$L = f\,\frac{nN_1N_2}{A}.$$

Spinors

$$\not{p}u = mu$$
$$\bar{u}\not{p} = \bar{u}m$$
$$\not{p}v = -mv$$
$$\bar{v}\not{p} = -\bar{v}m$$
$$\sum_s u\bar{u} = \not{p} + m$$
$$\sum_s v\bar{v} = \not{p} - m$$

External Fermion Lines

$u(p,s)$ $\quad\bar{u}(p,s)$ $\quad\bar{v}(p,s)$ $\quad v(p,s)$

Cross-Sections and Decay Rates

$$\Gamma = \frac{D}{2M} \qquad \sigma = \frac{D}{4\left|E_2\mathbf{p}_1 - E_1\mathbf{p}_2\right|}$$
$$D(\text{two}) = \frac{p}{16\pi^2 E_{\text{cm}}}\int \left|i\mathcal{M}\right|^2 d\Omega$$
$$D(\text{three}) = \frac{1}{8(2\pi)^5}\int dE_1\, dE_2\, d\Omega_1\, d\phi_{12} \left|i\mathcal{M}\right|^2$$