Particle Physics, Fall 2012 **Final Exam** December 11, 2012

Part I: Short Answer [100 points]
For each of the following, give a short answer (2-3 sentences, or a formula). [5 points each]

1. [This one might be hard for Sam]: What is your name? ______________________________

2. The cross section ratio $\sigma\left(e^+e^- \to \text{hadrons}\right)/\sigma\left(e^+e^- \to \mu^+\mu^-\right)$ is usually about flat, but occasionally undergoes steps up, where the "constant" value increases to a new, higher constant. What is causing those steps up?

3. Suppose you had calculate the amplitude for some quantum mechanical process involving photons, say $X \to Y\gamma$, and it takes the form $i\mathcal{M} = f^{\alpha}\varepsilon_{\alpha}^{*}$, where f^{α} is something complicated, and ε_{α} is the polarization of the photon. What could you do to check if your answer is gauge invariant?

4. Isospin is an abbreviation of "isotopic spin." What sort of isotopes can be related by isospin? For example, can it tell you how the energy levels of ^{4}He nuclei relate to energy levels of ^{238}U nuclei?

5. Which of the isospin or SU(3) flavor is a better approximation to reality? Is either of them a perfect symmetry? Give an argument for why they are imperfect, or not.

6. We used SU(3) twice in this class, and called them $SU(3)_F$ and $SU(3)_C$. Suppose I had a red up quark. What sort or particle might $SU(3)_F$ relate it to? What sort of particle might $SU(3)_C$ relate it to?

7. The $\left|\Sigma^+\right\rangle$ is a strangeness -1 baryon with charge +1. What is its quark content? Assume it contains only some subset of the three lightest quarks.

8. Free quarks have never been discovered. What happens, qualitatively, if you try to separate the quarks, say, in a meson, like $\left|\pi^+\right\rangle = \left|u\bar{d}\right\rangle$?

9. A $\left|\Xi^{*0}\right\rangle$ spin $\frac{3}{2}$ baryon with spin $S_z = +\frac{3}{2}$ would have its spin and quark content described as $\left|\Xi^{*0}, +\frac{3}{2}\right\rangle = \frac{1}{\sqrt{3}}\left(\left|u\uparrow, s\uparrow, s\uparrow\right\rangle + \left|s\uparrow, u\uparrow, s\uparrow\right\rangle + \left|s\uparrow, s\uparrow, u\uparrow\right\rangle\right)$. But this appears to be completely symmetric, yet it involves only fermions. How is this apparent discrepancy resolved?

10. How many colors are there for the electron, electron neutrino, up quark, down quark, gluon, and photon?

11. As you go to higher energies, it is known that the electromagnetic fine structure constant α gets stronger. Is this true of the strong coupling α_S as well?

12. The first theory of weak interactions we came up was the Fermi theory, with a coupling G_F which led to Feynman amplitudes like $i\mathcal{M} \sim G_F \left[\bar{u}\gamma^{\mu}\left(1-\gamma_5\right)v\right]\left[\bar{u}'\gamma_{\mu}\left(1-\gamma_5\right)v'\right]$. Why did we ultimately abandon this theory?

13. Why are weak interactions weak? In other words, why are processes like $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ so slow?

14. What is the difference between charged current and neutral current weak interactions? What particles are responsible for each of them?

15. What is the gauge group of the standard model?

16. Naively, you would think there is a an “up quark” field, with both left- and right- handed parts, and a “down quark” field, with both left- and right-handed quarks. But if it weren’t for symmetry breaking, these associations would be meaningless. What are the three actual fields that account for the up quark and down quark?

17. Because the Higgs field is a complex doublet, you would think that there would be four degrees of freedom, and hence four distinct particles. But only a single degree of freedom (the Higgs boson) has been discovered, and this is all that is expected. What happened to those other degrees of freedom?

18. It is known experimentally that there is *CP* violation. Where in the standard model can we find a source of *CP* violation?

19. The most common decay of the Higgs is $H \to b\bar{b}$. Yet this decay was not the (primary) one that was used at the LHC to detect the Higgs. Why is this decay difficult to distinguish?

20. When we calculate cross-sections like $\nu_e e^- \to \nu_e e^-$, we sum over the final spins, average over the initial electron spin, and <u>sum</u> over the initial neutrino spin. Why is this the right thing to do?

Part II: Calculation [200 points]
Each problem has its corresponding point value marked. Solve the equations on separate paper.

21. [20] Draw the Feynman diagrams relevant for electron-electron scattering, $e^-(p_1)e^-(p_2) \to e^-(p_3)e^-(p_4)$. Write the Feynman amplitude, including the correct relative sign. Assume you are at low enough energy that only QED contributions are important.

22. [10] In SU(3)$_F$ notation, the Ξ^0 particle is $|\Xi^0\rangle = |B_3^2\rangle$. Find the effects of all six $T_{i\to j}$ operators on $|\Xi^0\rangle$.

23. [15] Draw all relevant tree-level Feynman diagrams for a pair of gluons to collide and make a quark/anti-quark pair, $g(q_1)g(q_2) \to q(p_1)\bar{q}(p_2)$. Correctly label any intermediate momenta that arise. You don't have to calculate the amplitude.

24. [20] The three pions (π^-, π^0, π^+) form an isospin triplet, $I = 1$, for which we have

$$I_+ = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}, \quad I_- = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}.$$

(a) Work out $I_+|\pi\rangle$ and $I_-|\pi\rangle$ for all three particles.
(b) The ω is an isospin singlet, so $I_a|\omega\rangle = 0$. If isospin is a perfect symmetry, argue that $\langle\pi^+\pi^0|I_+\mathcal{H}|\omega\rangle = 0$.
(c) Show from part (b) that there is a simple relationship between $\langle\pi^+\pi^-|\mathcal{H}|\omega\rangle$ and $\langle\pi^0\pi^0|\mathcal{H}|\omega\rangle$.

25. [25] Consider the hypothetical process for production of top quarks by colliding neutrinos with positrons,

$$e^+(p_1)\nu_e(p_2) \to t(p_3)\bar{f}(p_4)$$

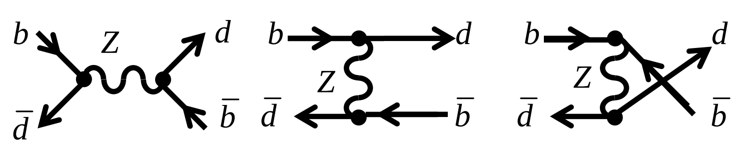
where f represents some sort of final state fermion
(a) Which possible fermions could it be? There should be multiple correct answers. Which fermion is most likely to be produced?
(b) Draw the relevant Feynman diagram.
(c) Write the Feynman amplitude for this process. For the moment, let q be the momentum of any intermediate particle that you need for the process.
(d) Write q in two ways in terms of the p_i's.
(e) Your propagator may contain terms that look something like $q_\mu q_\nu / M_X^2$. Argue that if we treat the positron and neutrino as massless, this term actually doesn't contribute.

26. [30] Bottom quarks can be produced via the process $e^+(p_1)e^-(p_2) \to b(p_3)\bar{b}(p_4)$. Assume we are working at sufficiently high energies that we need to consider all processes, not just QED.

(a) Show that there are three tree diagrams with different intermediate particles that contribute to this process in the standard model.
(b) Argue that one of them is essentially irrelevant.
(c) Write the Feynman amplitude for the other two

27. [35] Calculate the decay rate $H \to c\bar{c}$. For definiteness, use $m_H = 126\text{ GeV}$, $m_c = 1.29\text{ GeV}$, and $v = 246\text{ GeV}$. Get both a formula and a number (in MeV) for the process. You may treat the charm quark as massless compared to the Higgs (but don't get zero for the final answer). Don't forget about colors.

28. [20] The B^0 meson is a bottom – anti-down combination, $b\bar{d}$. It is known experimentally that it can spontaneously change into its anti-particle, $\bar{b}d$.



(a) Explain why the tree-level diagrams sketched at right do not contribute.
(b) Draw at least one one-loop diagram that does contribute to this process

29. [25] Suppose that the standard model is right, except that (i) the charges are actually slightly off, and (ii) there are right-handed neutrinos, so that the left-handed fermion content of the theory is given by

$$3\left[\left(3,2,\tfrac{1}{6}+\delta\right)\oplus\left(\bar{3},1,-\tfrac{2}{3}-\delta\right)\oplus\left(\bar{3},1,\tfrac{1}{3}-\delta\right)\oplus\left(1,2,-\tfrac{1}{2}-3\delta\right)\oplus\left(1,1,1+3\delta\right)\oplus\left(1,1,3\delta\right)\right]$$

(a) Find the charge of the left-handed up and down quarks using the $\left(3,2,\tfrac{1}{6}+\delta\right)$, and the charge of the neutrino and electron using $\left(1,2,-\tfrac{1}{2}-3\delta\right)$.

(b) Find the charge of the proton, neutron and hydrogen atom.
(c) Check if the anomalies $\sum Y$ and $\sum YT_3^2$ vanish.

Useful Formulas and Identities

Units and Constants

$$c = \hbar = \varepsilon_0 = \mu_0 = 1$$
$$1\text{ s} = 3\times10^8\text{ m}$$
$$1\text{ kg} = 5.6\times10^{26}\text{ GeV}$$
$$197\,\text{MeV}\cdot\text{fm} = 1$$
$$1\text{b} = 100\text{ fm}^2$$
$$m_e = 0.51100\text{ MeV}$$
$$m_p = 938.27\text{ MeV}$$
$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137.036}$$
$$\alpha_S(M_Z) = \frac{g_S^2}{4\pi} \approx 0.1184$$
$$G_F = 1.166\times10^{-5}\text{ GeV}^{-2}$$
$$v = 246\text{ GeV}$$
$$\sin^2\theta_W = 0.2312$$

Metric Prefixes

T	10^{12}
G	10^{9}
M	10^{6}
k	10^{3}
m	10^{-3}
μ	10^{-6}
n	10^{-9}
p	10^{-12}
f	10^{-15}

Dirac Matrices
(chiral representation)

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$
$$\boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}$$
$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Dirac Properties

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$
$$\{\gamma^\mu, \gamma_5\} = 0$$
$$\gamma_5\gamma_5 = 1$$
$$\overline{\Gamma} \equiv \gamma^0\Gamma^\dagger\gamma^0$$
$$\overline{\gamma}^\mu \equiv \gamma^\mu$$
$$\overline{\gamma}_5 = -\gamma_5$$

The Metric

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

More Dirac Identities

$$\gamma^\mu\gamma_\mu = 4$$
$$\gamma^\mu\gamma^\alpha\gamma_\mu = -2\gamma^\alpha$$
$$\gamma^\mu\gamma^\alpha\gamma^\beta\gamma_\mu = 4g^{\alpha\beta}$$
$$\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\nu\gamma_\mu = -2\gamma^\nu\gamma^\beta\gamma^\alpha$$

☺

Dirac Trace Identities

$$\text{Tr}\left(\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_{2N+1}}\right) = \text{Tr}\left(\gamma_5\gamma^{\mu_1}\gamma^{\mu_2}\cdots\gamma^{\mu_{2N+1}}\right) = 0$$
$$\text{Tr}(1) = 4 \qquad \text{Tr}\left(\gamma^\mu\gamma^\nu\right) = 4g^{\mu\nu}$$
$$\text{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\right) = 4\left(g^{\mu\nu}g^{\alpha\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\alpha}g^{\nu\beta}\right)$$
$$\text{Tr}(\gamma_5) = \text{Tr}\left(\gamma_5\gamma^\alpha\gamma^\beta\right) = 0$$
$$\text{Tr}\left(\gamma_5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\right) = -4i\varepsilon^{\mu\nu\alpha\beta}$$

Cross-Sections and Decay Rates

$$\Gamma = \frac{D}{2M} \qquad \sigma = \frac{D}{4\left|E_2\mathbf{p}_1 - E_1\mathbf{p}_2\right|}$$
$$D(\text{two}) = \frac{p}{16\pi^2 E_{\text{cm}}}\int\left|i\mathcal{M}\right|^2 d\Omega$$
$$D(\text{three}) = \frac{1}{8(2\pi)^5}\int dE_1\, dE_2\, d\Omega_1\, d\phi_{12}\left|i\mathcal{M}\right|^2$$

Spinors and Polarizations

$$\not{p}\,u(p,s) = mu(p,s)$$
$$\not{p}\,v(p,s) = -mv(p,s)$$
$$\sum_s u(p,s)\overline{u}(p,s) = \not{p} + m$$
$$\sum_s v(p,s)\overline{v}(p,s) = \not{p} - m$$
$$q\cdot\varepsilon(q,\lambda) = 0$$
$$\varepsilon^*(q,\lambda)\cdot\varepsilon(q,\tau) = \delta_{\lambda\tau}$$

massless:

$$\sum_\lambda \varepsilon^{*\mu}(q,\lambda)\varepsilon^\nu(q,\lambda) = -g^{\mu\nu}$$

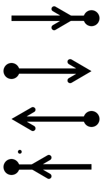
massive:

$$\sum_\lambda \varepsilon^{*\mu}(q,\lambda)\varepsilon^\nu(q,\lambda) = -g^{\mu\nu} + q^\mu q^\nu/M^2$$

Feynman Rules for the Standard Model

External lines

Fermions (any) | Spin -1 (any)

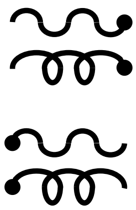


$u(p,s)$

$\bar{u}(p,s)$

$\bar{v}(p,s)$

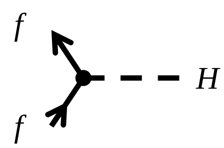
$v(p,s)$



$\varepsilon_\mu(q,\sigma)$

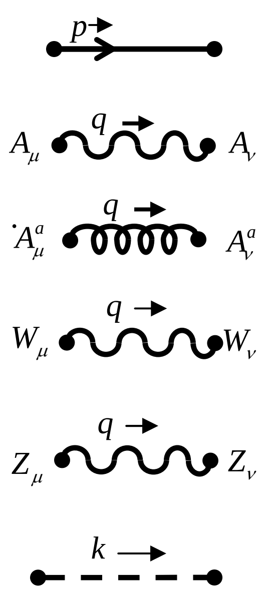
$\varepsilon^*_\mu(q,\sigma)$

Fermion-Higgs coupling



$-i\dfrac{m_f}{v}$

Propagators



$$\frac{i\left(\not p + m\right)}{p^2 - m^2} = \frac{i}{\not p - m}$$

$$-\frac{ig_{\mu\nu}}{q^2}$$

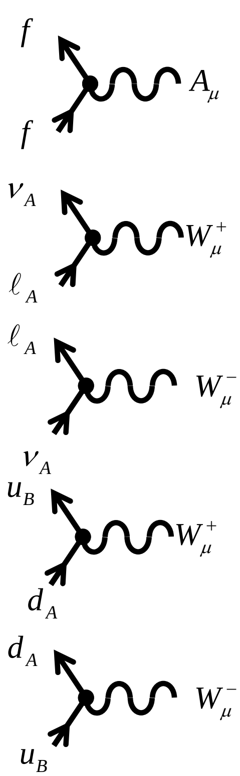
$$-\frac{ig_{\mu\nu}}{q^2}$$

$$\frac{i\left(-g_{\mu\nu} + q_\mu q_\nu / M_W^2\right)}{q^2 - M_W^2}$$

$$\frac{i\left(-g_{\mu\nu} + q_\mu q_\nu / M_Z^2\right)}{q^2 - M_Z^2}$$

$$\frac{i}{k^2 - M_H^2}$$

Fermion-gauge couplings



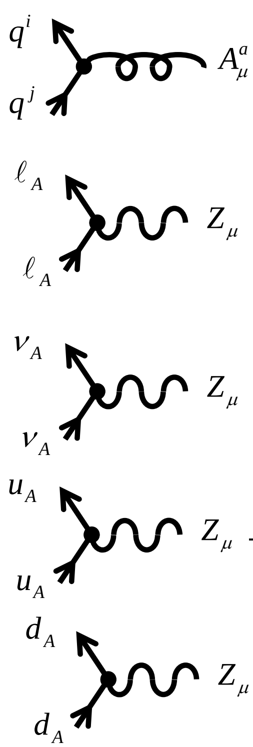
$$-ieQ\gamma^\mu$$

$$-i\frac{e\gamma^\mu\left(1-\gamma_5\right)}{2\sqrt{2}\sin\theta_W}$$

$$-i\frac{e\gamma^\mu\left(1-\gamma_5\right)}{2\sqrt{2}\sin\theta_W}$$

$$-i\frac{e\gamma^\mu\left(1-\gamma_5\right)}{2\sqrt{2}\sin\theta_W}V_{BA}$$

$$-i\frac{e\gamma^\mu\left(1-\gamma_5\right)}{2\sqrt{2}\sin\theta_W}V^*_{BA}$$



$$-ig_s\left(T_a\right)^i{}_j\,\gamma^\mu$$

$$\frac{ie\gamma^\mu\left(1-4\sin^2\theta_W-\gamma_5\right)}{4\sin\theta_W\cos\theta_W}$$

$$-\frac{ie\gamma^\mu\left(1-\gamma_5\right)}{4\sin\theta_W\cos\theta_W}$$

$$-\frac{ie\gamma^\mu\left(1-\frac{8}{3}\sin^2\theta_W-\gamma_5\right)}{4\sin\theta_W\cos\theta_W}$$

$$\frac{ie\gamma^\mu\left(1-\frac{4}{3}\sin^2\theta_W-\gamma_5\right)}{4\sin\theta_W\cos\theta_W}$$