Physics 744 - Field Theory

Homework Set 7

1. In the $\psi^*\psi\phi$ theory, consider the tree-level diagrams (no loops) contributing to the scattering $\psi(\mathbf{p}_1)\psi^*(\mathbf{p}_2)\to\psi(\mathbf{p}_1')\psi^*(\mathbf{p}_2')$.
   (a) Draw all (two) Feynman diagrams that contribute to this process and label the intermediate momenta. Write the corresponding Feynman amplitude.
   (b) Work out the differential cross-section in the center of mass frame. You may write your answer in terms of the energies E of any one of the particles or the magnitude of the three-momentum p as needed. Let θ represent the angle between the initial and final momenta of the ψ's.
   (c) Are there any subtleties having to do with final momenta? Find the total cross-section.

2. We wish to work out the one loop contribution to the propagator $\pi\left(p^2\right)$ for the ψ particle in the $\psi^*\psi\phi$ theory, using dimensional regularization.
   (a) Draw the relevant one loop diagram and write an expression for the Feynman amplitude
   (b) Combine the denominators using Feynman parameters. Shift the integral to make it spherically symmetric.
   (c) Regulate the integral using dimensional regularization in $d=4-2\varepsilon$ dimensions. Perform the momentum integrals.
   (d) Multiply out all the factors, keeping terms of $\mathcal{O}\left(\varepsilon^{-1}\right)$ and $\mathcal{O}(1)$, but dropping lower terms. You may leave one Feynman parameter undone.
   (e) Convince yourself, and me, that $\pi\left(m^2\right)$ is always real. Hence there is no problem calculating the counterterm.

2. We wish to work out the one loop contribution to the propagator $\pi\left(p^2\right)$ for the ϕ particle in the $\psi^*\psi\phi$ theory, using dimensional regularization. This was done in class, but I want you to redo it using dimensional regularization.
   (a-d) Same as previous problem.
   (e) Convince yourself, and me, that $\pi\left(M^2\right)$ is real if $M<2m$. If $M>2m$, find the imaginary part, and compare its value to the decay rate for the ϕ, given by $\Gamma=\gamma^2\sqrt{M^2-4m^2}\big/16\pi M^2$.

Useful formula: $$\lim_{\varepsilon\to 0}\left[\ln\left(x-i\varepsilon\right)\right]=\begin{cases}\ln x & \text{if } x>0,\\ -i\pi+\ln|x| & \text{if } x<0.\end{cases}$$