Physics 741 – Graduate Quantum Mechanics 1

Solutions to Chapter 6

6.1 [10] For the finite square well in section C, we showed that (6.24) is satisfied for the even wave functions. Repeat this derivation for the odd wave functions; i.e., derive (6.25).

We know from class notes that in the three regions, the solution takes the form

$$\psi_I(x) = Ae^{\beta x}$$
$$\psi_{II}(x) = B\cos(kx) + C\sin(kx)$$
$$\psi_{III}(x) = De^{-\beta x}$$

We are interested in the odd parity bound states. Since parity relates regions I and III to each other, and region II to itself, this implies $-\psi_I(x) = \psi_{III}(-x)$ and $\psi_{II}(-x) = -\psi_{II}(x)$. We therefore have $A = -D$ and $B = 0$.

We now wish to match boundary conditions. We will choose to do so at $x = a$, where we follow the notes to yield

$$\psi_{II}(a) = \psi_{III}(a) \quad \Rightarrow \quad C\sin(ka) = D\exp(-\beta a)$$
$$\psi'_{II}(a) = \psi'_{III}(a) \quad \Rightarrow \quad kC\cos(ka) = -\beta D\exp(-\beta a)$$

Dividing the second equation by the first, we find

$$k\cot(ka) = -\beta$$

Using equations (6.22) and (6.23), it is easy to see that

$$k^2 + \beta^2 = 2mV_0\hbar^{-2} \quad \Rightarrow \quad \beta = \sqrt{2mV_0\hbar^{-2} - k^2}$$

Plugging this in and dividing by $-k$, we find

$$-\cot(ka) = \sqrt{\frac{2mV_0}{\hbar^2 k^2} - 1}$$

This is equation (6.25). Solutions of this equation can then be substituted into (6.23) to get the energy eigenvalues.