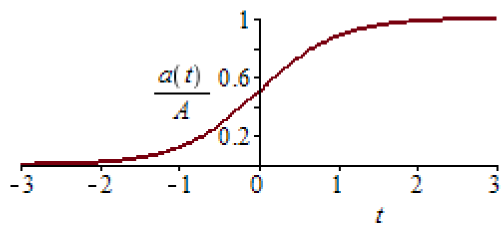
PHY 742 Spring 2025 Second Exam Name ____________________

Each question is worth 20 points. Some possibly useful formulas appear below or on the handout

1. A particle of mass μ with momentum $\hbar k$ scatters from a potential of the form $V(\mathbf{r}) = V_0 e^{-Ar^2}$. Using the first Born approximation, find the differential and total cross-section. I recommend working in Cartesian coordinates.

2. A particle lies in a 1d moving harmonic oscillator with potential $V(x,t) = \frac{1}{2} m\omega^2 \left[x - a(t)\right]^2$, where $a(t)$ is a smoothly increasing function from $a(t = -\infty) = 0$ to $a(t = \infty) = A$. It is initially in the ground state with wave function $\psi_0(x) = (m\omega/\pi\hbar)^{1/4} e^{-m\omega x^2/2\hbar}$. What is the probability that it is still in the ground state at $t = \infty$ if the increase is (a) adiabatic, (b) sudden?



3. A spin-½ particle is in a magnetic field in the *z*-direction has two spin states $|\pm\rangle$ with energies $E_\pm = \mp\frac{1}{2}\hbar\omega_B$. It is initially in the spin up $|+\rangle$ state. It is subjected to a brief perturbation of the form $W(t) = \lambda S_x e^{-At^2}$. What is the probability that it flips to the $|-\rangle$ state? Recall that $S_x|\pm\rangle = \frac{1}{2}\hbar|\mp\rangle$.

4. A system of pure photons is in the state $|\Psi\rangle = A\left(4|8,\mathbf{q},\tau\rangle - 3i|9,\mathbf{q},\tau\rangle\right)$, where $\mathbf{q} = q\hat{\mathbf{x}}$ and $\boldsymbol{\varepsilon}_{\mathbf{q}\tau} = \hat{\mathbf{y}}$. Find the normalization A and the expectation value of the magnetic field $\langle\Psi|\mathbf{B}(\mathbf{r})|\Psi\rangle$.

5. An electron of mass *m* is in the $|3,2,1\rangle$ state of the 3D symmetric harmonic oscillator with angular frequency ω. In the dipole approximation, which states can it decay to, and what are the corresponding rates?

Possibly Helpful Formulas:

Spontaneous Decay

$$\Gamma = \frac{4\alpha}{3c^2}\omega_{IF}^3 \left|\mathbf{r}_{FI}\right|^2$$

1st Born Approximation

$$\frac{d\sigma}{d\Omega} = \frac{\mu^2}{4\pi^2\hbar^4}\left|\int d^3\mathbf{r}\, V(\mathbf{r}) e^{-i\mathbf{K}\cdot\mathbf{r}}\right|^2$$

$$\mathbf{K}^2 = 2k^2(1-\cos\theta)$$

1D harmonic oscillator:

$$V(x) = \tfrac{1}{2}m\omega^2 x^2$$

$$X = \sqrt{\frac{\hbar}{2m\omega}}\left(a + a^\dagger\right)$$

$$a|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

Magnetic Field Operator

$$\mathbf{B}(\mathbf{r}) = \sum_{\mathbf{k},\sigma}\sqrt{\frac{\hbar}{2\varepsilon_0 V\omega_k}}\, i\mathbf{k}\times\left(a_{\mathbf{k}\sigma}\boldsymbol{\varepsilon}_{\mathbf{k}\sigma}e^{i\mathbf{k}\cdot\mathbf{r}} - a^\dagger_{\mathbf{k}\sigma}\boldsymbol{\varepsilon}^*_{\mathbf{k}\sigma}e^{-i\mathbf{k}\cdot\mathbf{r}}\right)$$

Time-dependent Pert. Theory

$$S_{FI} = \delta_{FI} + \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt\, W_{FI}(t)\, e^{i\omega_{FI}t} + \cdots$$

Trigonometry: $e^{i\theta} + e^{-i\theta} = 2\cos\theta, \quad e^{i\theta} - e^{-i\theta} = 2i\sin\theta.$

Integrals: $\int_{-\infty}^{\infty} e^{-Ax^2+Bx} dx = \sqrt{\frac{\pi}{A}} e^{B^2/4A}, \quad \int_0^{\infty} x^n e^{-\alpha x} dx = n!\alpha^{-(n+1)}.$

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