MTH 111 Calculus I

Second Hour Exam

October 25, 2000 Elmer K. Hayashi

Mathematics is loved by many, disliked by a few, admired and respected by all. Because of their immense power and reliability, mathematical methods inspire confidence in persons who comprehend them and awe in those who do not.

-Hollis R. Cooley.

Budget your time carefully. Do not spend too much time on one problem. On the other hand, remember that in most cases showing how you get the answer is worth more points than the answer itself.

- 1. (a) What is the range of the natural exponential function  $f(x) = e^x$ ? 17 pts
  - (b) What is the domain of the inverse tangent function  $f(x) = \arctan(x)$ ?
  - (c) Find the formula for the inverse function of  $f(x) = \frac{3x+2}{x-1}$ .
- 2. Find the derivative of each of the following functions showing significant intermediate steps. 19 pts

(a) 
$$y = \arcsin(4x)$$
. (b)  $y = \frac{x^3 e^{2x}}{\sqrt{x^2 + 1}}$  (c)  $y = 3^{x^2} + x^5$ 

3. Evaluate each of the following integrals showing the substitution used and important inter- 21 pts mediate steps.

(a) 
$$\int x^2 e^{2x^3} dx.$$
 (b)  $\int \frac{x^2 + 1}{x^3} dx.$  (c)  $\int_0^{\sqrt[4]{3}} \frac{x}{1 + x^4} dx.$ 

- 4. Find the exact value of each of the following expressions:
  - (a)  $\operatorname{arccos}\left(-\frac{1}{\sqrt{2}}\right)$  (b)  $\operatorname{sin}\left(2\arctan\left(\frac{3}{4}\right)\right)$ .
- 5. Find the value of each of the following limits giving justification for your conclusions as 26 pts appropriate.
  - (a) Find  $\lim_{x \to \infty} \frac{3e^x + 5}{e^x 2}$ . (b) Find  $\lim_{x \to -\infty} \frac{3e^x + 5}{e^x - 2}$ . (c) Find  $\lim_{x \to -\infty} \frac{2x + 5}{\sqrt{3x^2 + 4x + 1}}$ . (d) Find  $\lim_{x \to \infty} \arctan(x)$ .
- 6. Write a definite integral whose value gives the volume of the solid obtained by rotating about 7 pts the y-axis the region under the curve  $y = \frac{1}{1+x^4}$  from x = 0 to x = 1. Do not evaluate the integral.

10 pts