Mathematics is the life supreme. The life of the gods is mathematics. All divine messengers are mathematicians. Pure mathematics is religion. Its attainment requires a theophany.

The first six integrals were on a MTH 111 exam given in November, 2000. The other questions were added to review material that we have additionally covered.

1. Evaluate each of the following integrals:

(a)
$$\int x^3 \ln(x) \, dx;$$

(b) $\int \frac{x^2 + 4x + 1}{(x+2)(x-1)^2} \, dx;$
(c) $\int \frac{1}{x^3\sqrt{x^2-9}} \, dx;$
(d) $\int \frac{x^3 + 3x + 1}{x^2 + 1} \, dx;$
(e) $\int_0^{\frac{\pi}{2}} x \cos(x) \, dx;$
(f) $\int \frac{(x-2)^3}{\sqrt{4x-x^2}} \, dx$

2. A rectangular swimming pool is 30 ft. wide and 50 ft. long. The depth of the water in the pool is constant from side to side, but varies along the length of the pool as shown in the chart.

position in ft.	0	5	10	15	20	25	30	35	40	45	50
depth in ft.	6.0	8.2	9.1	9.9	10.5	11.0	11.5	11.9	12.3	12.7	13.0

- (a) Approximate the volume of water in the pool using the Trapezoid method with n = 5.
- (b) Approximate the volume of water in the pool using the Midpoint method with n = 5.
- (c) Approximate the volume of water in the pool using Simpson's method with n = 10.
- 3. Consider the curve $y = \sqrt[3]{x}$ from x = 1 to x = 8.
 - (a) Find the length of the arc of the curve by setting up the integral, and then using Simpson's Method with n = 16 to approximate the integral.
 - (b) If you did part (a) by integrating with respect to x, do part (a) again by integrating with respect to y, or vice versa. The arc length should be the same either way, but the integral with respect to y gives a slightly better approximation.
 - (c) Find an estimate for the error in the approximation in part (a) or (b). Note that it is the second derivative of the integrand that you must use to find K, not the second derivative of the given function.
 - (d) Find the surface area generated by rotating the given arc about the x-axis. Set up the integral, and then use Simpson's Method with n = 16 to approximate the integral.