MTH 112
Calculus II

First Hour Exam

September 26, 2001
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Mathematics is loved by many, disliked by a few, admired and respected by all. Because of their immense power and reliability, mathematical methods inspire confidence in persons who comprehend them and awe in those who do not.

–Hollis R. Cooley.

1. Find a formula for the general term a_n of the following sequence, and find the limit of the sequence. Justify your conclusions.

$$\left\{\frac{-\ln(2)}{3}, \frac{\ln(3)}{8}, \frac{-\ln(4)}{13}, \frac{\ln(5)}{18}, \frac{-\ln(6)}{23}, \ldots\right\}$$

2. Determine whether each of the given series is convergent or divergent. Justify your conclusions and show any necessary computations.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{5n+4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^3-2n}{4n^5+1}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{7n}{1+6n}\right)^n$

(f) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

3. Determine whether each given series is conditionally convergent, absolutely convergent, or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n}{2n^2+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)\arctan(n)}{n^3}$

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The solutions to these problems are due at the beginning of class on Friday, September 28, 2001. You may consult your textbook, and use Maple or a calculator to assist you with the computations, but you may not discuss these problems with anyone until all solutions have been turned in. Your answers may be written on this page or on separate sheets of paper as you like.

1. Find the sum of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$ accurate to five decimal places. Justify your conclusions.

2. Calculate the thirtieth partial sum, s_{30}, of the series $\sum_{n=1}^{\infty} \frac{3}{n^4}$ as an approximation for the total sum of the series. Estimate the error in using the thirtieth partial sum to approximate the total sum of the given series. Justify your conclusions.

3. Consider the recursive sequence defined by

$$a_1 = 2; \qquad\qquad a_{n+1} = \sqrt{5 + 2a_n}, \quad n \geq 1.$$

Use mathematical induction to prove that the sequence $\{a_n\}$ is an increasing sequence.