Math 112
Calculus II

Second Hour Exam

October 30, 2001
Elmer K. Hayashi

I love mathematics not only for its technical applications, but principally because it is beautiful; because man has breathed his spirit of play into it, and because it has given him his greatest game–the encompassing of the infinite.

–Rózsa Péter, *Playing with Infinity*, 1962.

1. Find the radius of convergence and the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+3)^n}{2^n \sqrt{n}}$$

2. (a) What is the Maclaurin's series for $f(x) = \cos(x)$?

   (b) Use the answer to part (a) to obtain the Maclaurin's series for $\cos(x^2)$.

   (c) Use the answer to part (b) to obtain a series for $\displaystyle\int_0^1 \cos(x^2)\, \mathrm{d}x$

   (d) What is the interval of convergence for the series in part (c)? Please explain.

   (e) Use the answer to part (b) to find $\displaystyle\lim_{x \to 0} \frac{x^4}{1 - \cos(x^2)}$

3. (a) Find the Maclaurin's series for $f(x) = \dfrac{1}{1+x^3}$, and determine its radius of convergence.

   (b) Use the answer to part (a) to find the Maclaurin's series for $f'(x)$.

   (c) Use the answer to part (b) to find the Maclaurin's series for $\dfrac{x^2}{(1+x^3)^2}$, and determine its radius of convergence.

   (d) Use the answer to part (a) to find the value of $f^{(15)}(0)$. You do not need to compute large factorials.

4. What region in three space is described by the following inequality?

$$x^2 + y^2 + z^2 < -4x + 4y + 2z$$

5. Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{2^{2n} n!}$$

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You may use your book, your notes, Maple and/or a calculator to do this take home portion of the exam. However, you should not discuss this with anyone until you have completed it, and turned it in on Wednesday, Nov. 1st.

1. (a) Find the Taylor polynomial of degree 5 for $f(x) = \sin(x)$ about $x = \frac{\pi}{4}$ ($a = \frac{\pi}{4}$).
   (b) Use the Taylor polynomial found above to approximate $\sin(0.6)$.
   (c) Use Taylor's inequality to estimate the error in your approximation of $\sin(0.6)$. To how many decimal places will your approximation of $\sin(0.6)$ be accurate?

2. Given the line $L: \quad \dfrac{x-4}{2} = \dfrac{y+2}{1} = \dfrac{z-3}{-3}$, and the point $P(5, -1, 5)$.
   (a) Find an equation of the plane that is perpendicular to L and contains the point P.
   (b) Find an equation of the plane that contains the line L and the point P.
   (c) Show that the point $Q(10, 1, -6)$ is on the line L.
   (d) Let L_2 be the line that contains P, intersects the line L, and is perpendicular to the line L. Find the distance from the point Q to line L_2. Hint: You don't need to find equations for L_2; all you need is a normal to L_2.

3. Given the planes $2x - 2y + 3z = 6$ and $5x + y - 3z = 3$.
   (a) Find the parametric equations of the line of intersection of the two planes.
   (b) Find the angle between the two planes to the nearest tenth of a degree.