

Like many great temples of some religions, mathematics may be viewed only from the outside by those uninitiated into its mysteries . . . understanding its methods is reserved for those who devote years to the study of mathematics.

—Andrew Gleason

1. Let $f(x, y) = \sqrt{x^2 - 6x + y^2 + 2y - 6}$. 12 pts
 - (a) What is the domain of the function $f(x, y)$?
 - (b) Sketch the domain of $f(x, y)$, or describe it carefully in words.
 - (c) What is the range of $f(x, y)$?
2. Let $g(x, y) = 2x^2 - 3y^2$ and let $h(x, y) = \frac{1}{2x^2 + 3y^2}$. 12 pts
 - (a) Explain the difference between the level sets of $g(x, y)$ and those of $h(x, y)$.
 - (b) Explain the difference in behavior between $g(x, y)$ and $h(x, y)$ as $(x, y) \rightarrow (0, 0)$.
 - (c) Explain the difference in behavior between $g(x, y)$ and $h(x, y)$ as $x^2 + y^2 \rightarrow \infty$.
3. Find each of the following limits, or show that the limit does not exist. 18 pts
 - (a) $\lim_{(x,y) \rightarrow (0,0)} \frac{(x+1)(y-2)}{(x+1)^2 + (y-2)^2}$.
 - (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y - y^3}{x^2 + y^2}$
4. Find the equation of the tangent plane to the surface $z = 2ye^{3xy}$ at the point $(0, 1, 2)$. 10 pts
5. Evaluate the iterated integral $\int_0^{\sqrt{\frac{\pi}{2}}} \int_0^{\sqrt{x}} y \cos(x^2) \, dy \, dx$ 10 pts
6. Sketch the region of integration and then change the order of integration for the following iterated integral. Assume that $f(x, y)$ is continuous throughout the region of integration. 10 pts
$$\int_0^1 \int_{2y^2}^{3-y} f(x, y) \, dx \, dy$$
7. If $f_y(1, 2)$ is the instantaneous rate of change of $f(x, y)$ with respect to y at $(1, 2)$ when $x = 1$ is held constant, then what is $f_{yx}(1, 2)$? 4 pts

You may use your book, your notes, Maple and/or a calculator to do this take home portion of the exam. However, you should not discuss this with anyone until you have turned it in on Friday, November 30, 2001.

1. Show that the following limit does not exist. 6 pts

$$\lim_{(x,y) \rightarrow (3,0)} \frac{(x-3)y}{(x-3)^2 + y^2}$$

2. Use differentials (Linearization) to approximate the amount of metal in a closed tin can with circular base having diameter 10 cm and height 12 cm if the thickness of the tin is 0.01 cm on the sides, 0.02 cm on top, and 0.03 cm on the bottom. 6 pts
3. Use the midpoint rule with 16 squares of equal size to approximate the double integral over $R = [0, 1] \times [0, 1]$ given below 6 pts

$$\iint_R \cos(\sqrt{x^2 + y^2}) \, dA$$

4. Set up the double integral to find the volume under the paraboloid $z = 3x^2 + 4y^2$ and over the region bounded by $y = x + 1$ and $x = 1 - y^2$. You do not need to evaluate this double integral. 6 pts
5. Problems 49 and 51 on page 997.