MTH 112 First Hour Exam February 13, 2001
Calculus II Elmer K. Hayashi

Mathematics must subdue the flights of our reason; they are the staff of the blind; no one can take a step without them; and to them and experience is due all that is certain in physics.

–Voltaire.

1. A sequence is defined by $a_n = r^n$, where r is a constant. For what values of r will the sequence converge? What is the limit? (No justification required.)

2. Determine whether each of the given series is convergent or divergent. Indicate the test you use and show any necessary computation.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+1}{5n^4-n+4}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+1}{3n^2+4}$

3. Determine whether each given series is conditionally convergent, absolutely convergent, or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

4. Find the sum of $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+5^n}{15^n}$, and explain why the given series is convergent.

5. Consider the recursive sequence defined by

$$a_1 = 1; \qquad a_{n+1} = \frac{a_n^2+2}{2a_n}, \quad n > 1.$$

   (a) Write the first three terms of this sequence.

   (b) You may assume the sequence to be monotone decreasing (for $n \geq 2$) and bounded and hence convergent. Find its limit.

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The solutions to these problems are due at the beginning of class on Wednesday, February 14, 2001. You may use Maple or a calculator to assist you with the computations.

1. Find the sum of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4}$ accurate to four decimal places. Justify your conclusions.

2. Calculate the fiftieth partial sum, s_{50}, of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2+cos(n)}{n^5}$ as an approximation for the total sum of the series. Estimate the error in using the fiftieth partial sum to approximate the total sum of the given series. Justify your conclusions.