Math 112 | Third Hour Exam | April 24, 2001
Calculus II | | Elmer K. Hayashi

Mathematics is the gate and key of the sciences. ...Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy.

–Roger Bacon

1. Let $g(x, y) = \frac{1}{x^2+y^2}$.

   (a) Describe the domain of the funciton $g(x, y)$.

   (b) Describe the level sets of the function $g(x, y)$.

   (c) Describe at least two other distinctive characteristics of the graph $z = g(x, y)$ that would help you distinguish it from the graphs of other functions.

2. Find each of the following limits, or show that the limit does not exist.

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{3\cos(x-2y)}{x^2-2y^2+1}$.

   (b) $\lim_{(x,y)\to(0,0)} \dfrac{x^3y}{x^2+y^2}$

3. Let $f(x, y) = \sqrt{50 - x^2 - 16y^2}$.

   (a) Find the linearization (tangent plane approximation) of $f(x, y)$ at the point $(3, 1)$.

   (b) Use the linearization to approximate $f(2.95, 1.05)$.

4. Let $D = \{(x, y)|0 \le x \le \frac{\pi}{2}, 0 \le y \le 1\}$.

   (a) Evaluate the double integral $\displaystyle\int\int_D x\cos(xy)\ \mathrm{d}A$

   (b) Find the average value of $f(x, y) = x\cos(xy)$ over D.

5. Evaluate the followinhg iterated integral by first reversing the order of integration.

$$\int_0^1 \int_{2x}^2 e^{y^2}\ \mathrm{d}y\ \mathrm{d}x$$

6. Set up, but do NOT evaluate, the double integral that gives the volume under the paraboloid $z = 3x^2 + y^2$, and above the region bounded by $y = x$ and $x = 2 - y^2$.

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You may use your book, your notes, Maple and/or a calculator to do this take home portion of the exam. However, you should not discuss this with anyone until you have turned it in.

1. Let $h(x, y) = \sqrt{16 - x^2 - 4y^2}$.
   (a) Describe and sketch the domain of the given function $h(x, y)$.
   (b) Sketch the graph of the function $z = h(x, y)$.

2. Find the following limit, or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$$

3. Use the midpoint rule with 256 squares of equal size to approximate the double integral over $R = [0, 1] \times [0, 1]$ given below

$$\int\int_R \sin(x^2 + y^2)\ \mathrm{d}A$$