

# Homework 3

## Numerical Linear Algebra

September 22, 2017

### 1 Problems for everybody

1. Problems 6.1, 6.4, 7.1
2. Recall, for any subspace  $V \subset \mathbb{R}^n$  that

$$V^\perp = \{\vec{v} \in \mathbb{R}^n : \forall \vec{w} \in V, \vec{v}^T \vec{w} = 0\}.$$

Let  $A \in \mathbb{R}^{n \times n}$ . Prove the following:

- (a)  $\text{null}(A^T) = (\text{range}(A))^\perp$ ,
- (b)  $\text{range}(A^T) = (\text{null}(A))^\perp$ ,
- (c)  $\text{null}(T) = (\text{range}(A^T))^\perp$ ,
- (d)  $\text{range}(T) = (\text{null}(A^T))^\perp$ .

### 2 Problem for math undergraduate and education majors

1. Suppose we want to solve  $A\vec{x} = \vec{b}$ . What condition on  $\vec{b}$  must be satisfied in order to guarantee the existence of a solution to this problem. **Hint:** Problem 2 above is an incredibly useful theorem that might be relevant to this problem.

### 3 Problem for math graduate students

1. Let  $A, B \in \mathbb{R}^n$  and suppose  $C = A + B$ . Let  $\sigma_a, \sigma_b, \sigma_c$  be the largest singular values of  $A, B$ , and  $C$  respectively. Form a conjecture about how  $\sigma_c$  relates to  $\sigma_a$  and  $\sigma_b$ . Prove this conjecture.
2. Problem 6.5