MST 351/651
Classwork #1

The goal of this classwork is to study a modified version of the harvesting of fish model we analyzed in class:

$$\dot{N} = rN\left(1 - \frac{N}{\kappa}\right) - H,$$

where $r, \kappa, H > 0$ are parameters. The problems with this model are at least threefold:

- This model allows N to become negative.
- The harvesting is independent of the number of fish. We would expect harvesting to decrease as the number of fish decreases to account for scarcity of the fish.
- Harvesting is not zero if there are zero fish.

The modified model we will consider is the following:

$$\dot{N} = rN\left(1 - \frac{N}{\kappa}\right) - H\frac{N}{A+N} = rN\left(1 - \frac{N}{\kappa}\right) - Hg(N), \tag{1}$$

where $A > 0$ is a constant.

1. Explain in practical terms what each parameter r, κ, H and A measure.

2. Sketch a graph of $g(N)$. Explain why $Hg(N)$ is a more realistic model of harvesting.

3. By introducing appropriate dimensionless variables x and τ show that Equation (1) can be expressed in the following dimensionless form:

$$\frac{dx}{d\tau} = x(1-x) - \frac{hx}{a+x}. \tag{2}$$

4. Determine the fixed points of Equation (2) as functions of a and h.

5. Show that the fixed point $x = 0$ is stable if $h > a$. (**Hint:** It is easier to do this using the analytic approach).

6. Show that three fixed points exist if

$$h < \frac{(a-1)^2}{4} + a.$$

7. Assuming

$$h < a < \frac{(a-1)^2}{4} + a,$$

sketch a phase portrait for Equation (2). What does this phase portrait imply in practical terms about the population of fish?

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10. For a fixed value of a, sketch the bifurcation diagram for Equation (2) with h as the bifurcation parameter. What does this bifurcation diagram tell you about this system?