Homework 8

Mathematical Modeling

Due: November 7, 2018

1 Problems for Everybody

1. Consider the following model for a drug prescription:

$$a_{n+1} = a_n - ka_n + b,$$

where a_n is the amount of a drug (in mg) in the bloodstream after administration of n dosages hourly.

   (a) Discuss the meaning of the model parameters k and b. What can you say about their size and sign?

   (b) Find the fixed points of the model and determine their stability.

   (c) Perform a cobwebbing analysis for this model. What happens to the amount of drug in the bloodstream in the long run? How does the result depend on the model parameters?

   (d) How should b be chosen to ensure that the drug is effective, but not toxic?

2. In this problem you can use Mathematica to help you with the calculations. You just need to include a copy of your code. Consider the discrete logistic equation:

$$x_{n+1} = rx_n(1 - x_n).$$

   (a) Compute $f^2(x)$, $f^3(x)$, and $f^4(x)$.

   (b) Find the fixed points of $f^2(x)$, $f^3(x)$, and $f^4(x)$. At what values of r, if any, does a 2-cycle, 3-cycle, or 4-cycle appear. For what values of r are these cycles stable? Unstable?

3. Consider the following discrete dynamical system

$$x_{n+1} = ax_n e^{-x_n} \text{ and } x_0 > 0.$$

where $a > 0$.

   (a) Find the fixed points and analyze their stability.

   (b) Show that for all n, $x_n \geq 0$.

   (c) Using Matlab, or some other software, create the orbital bifurcation diagram for this problem. Print and include a copy of this diagram with your homework. Compare and contrast this orbital bifurcation diagram with the one from the logistic equation.