

Math 205  
Classwork #8

October 13, 2021

1. In the following problems sketch the solution curves as functions of time  $t$  for the following differential equations. Be sure to calculate any inflection points and make sure your solution curves change concavity at the correct points.

(a)  $\frac{dx}{dt} = 4x^2 - 16$

(b)  $\frac{dx}{dt} = x - x^3$

$$(c) \frac{dx}{dt} = 1 + \frac{1}{2} \cos(x)$$

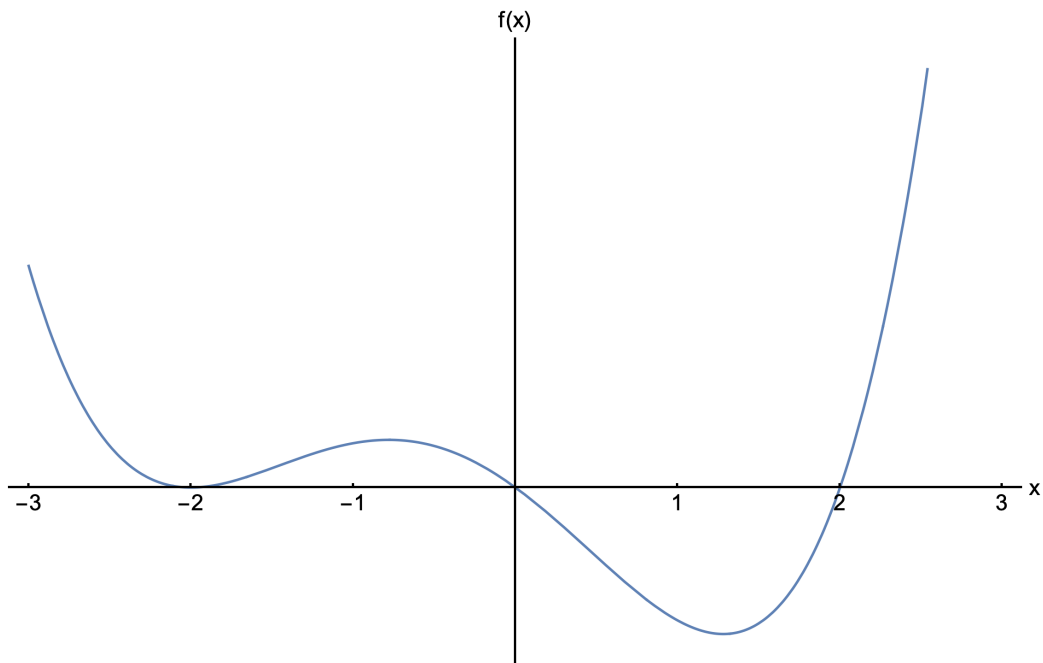
$$(d) \frac{dx}{dt} = 1 - 2 \cos(x)$$

$$(e) \frac{dx}{dt} = e^{-x} \sin(x)$$

2. Consider the differential equation

$$\frac{dx}{dt} = f(x),$$

where  $f(x)$  is plotted below.



- (a) On the figure indicate any fixed points, i.e. equilibrium points, for this differential equation.
  
- (b) On one axis, sketch the corresponding solutions curves  $x(t)$  for this problem. Your solution curves should contain all possible qualitatively different types of solution curves.

3. The curves  $x(t)$  illustrated below correspond to solution curves for the differential equation  $\frac{dx}{dt} = f(x)$ .

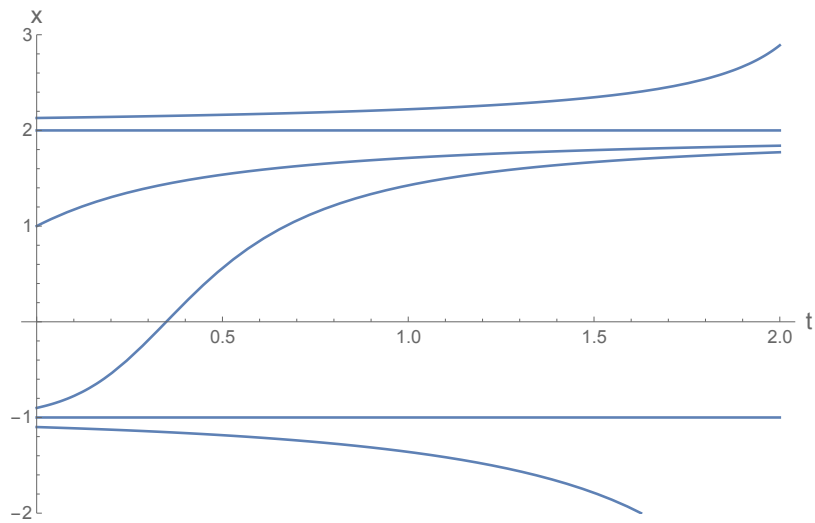


Figure 1:

- (a) Sketch a graph of  $f(x)$  that is consistent with the above figure.  
 (b) Give a formula for  $f(x)$  that is consistent with the above figure.

4. For each of (a)-(d) find an equation  $\frac{dx}{dt} = f(x)$  with the stated properties, or if there are no examples, explain why not. In each problem, assume that  $f$  is a smooth function, i.e. infinitely differentiable.
- (a) Every real number is a fixed point.
  - (b) Every integer is a fixed point, and there are no others.
  - (c) There are no fixed points.
  - (d) There are precisely 100 fixed points.