MST 205
Fall 2021
Exam #3
11/11/21

Name (Print): ______________________

The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Short answer questions:** Questions labeled as **"Short Answer"** can be answered by simply writing an equation or a sentence or appropriately drawing a figure. No calculations are necessary or expected for these problems.
- **Unless the question is specified as short answer, mysterious or unsupported answers might not receive full credit.** An incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	15	
3	20	
4	20	
5	15	
6	10	
Total:	100	

1. (20 points) Consider the following differential equation:

$$\frac{dy}{dx} + \sin(x)y^2 = 0.$$

(a) (5 points) **Short Answer:** For this differential equation, check all boxes that apply.

- [x] **First order**
- [ ] **Second order**
- [ ] **Linear**
- [x] **Nonlinear**
- [ ] **Constant coefficient**
- [x] **Non-constant coefficients**
- [x] **Homogeneous**
- [ ] **Non-homogeneous**

(b) (5 points) Show that $y = 0$ is a solution to this equation.

$\frac{d0}{dX} = 0 \qquad 0 + \sin(x)\, 0 = 0$ ✓

(c) (10 points) Find the general formula for other solutions to this equation.

$$\frac{1}{y^2}\frac{dy}{dx} = -\sin(x)$$

$$\Rightarrow -\frac{1}{y} = \cos(x) + c$$

$$\Rightarrow y = \frac{1}{C - \cos(x)}.$$

2. (15 points) Consider the following differential equation:

$$x\frac{dy}{dx} + x^2 y = 0.$$

(a) (5 points) **Short Answer:** For this differential equation, check all boxes that apply.

- [x] **First order**
- [ ] **Second order**
- [x] **Linear**
- [ ] **Nonlinear**
- [ ] **Constant coefficient**
- [x] **Non-constant coefficients**
- [x] **Homogeneous**
- [ ] **Non-homogeneous**

(b) (10 points) Find the general formula for solutions to this equation.

$$\frac{dy}{dx} + xy = 0$$

$$\frac{d}{dx}\left(e^{\frac{1}{2}x^2}y\right) = 0$$

$$\Rightarrow y = Ce^{-\frac{1}{2}x^2}$$

3. (20 points) Consider the following differential equation:

$$y''(x) + 2y'(x) + 2y(x) = 0.$$

(a) (5 points) **Short Answer:** For this differential equation, check all boxes that apply.

☐ **First order**	☑ **Second order**
☑ **Linear**	☐ **Nonlinear**
☑ **Constant coefficient**	☐ **Non-constant coefficients**
☑ **Homogeneous**	☑ **Non-homogeneous**

(b) (10 points) Find the general solution to this equation.

$y = c_1 e^x \cos(x) + c_2 e^x \sin(x)$

(c) (5 points) Use your general to solve this differential equation with the following initial conditions

$$y(0) = 1,$$
$$y'(0) = 0.$$

$\Rightarrow y(0) = 1 \Rightarrow c_1 = 1$

$y'(0) = 0 \Rightarrow c_2 = 0$

$y = e^x \cos(x).$

4. (20 points) Consider the following differential equation:

$$y''(x) - 2y(x) = 6e^{2x} - 4e^{-2x}.$$

(a) (5 points) **Short Answer:** For this differential equation, check all boxes that apply.

☐ **First order**	☑ **Second order**
☑ **Linear**	☐ **Nonlinear**
☑ **Constant coefficient**	☐ **Non-constant coefficients**
☐ **Homogeneous**	☑ **Non-homogeneous**

(b) (15 points) Find the general solution to this equation.

$$y = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x} + Ae^{2x} + Be^{-2x}.$$

5. (15 points) Consider the following differential equation:

$$x^2 y''(x) - 2y(x) = 0.$$

(a) (5 points) **Short Answer:** For this differential equation, check all boxes that apply.

- [ ] **First order**
- [x] **Second order**
- [x] **Linear**
- [ ] **Nonlinear**
- [ ] **Constant coefficient**
- [x] **Non-constant coefficients**
- [x] **Homogeneous**
- [ ] **Non-homogeneous**

(b) (5 points) By making a guess of the form $y_g(x) = x^p$ for some power p, derive an equation for p ~~for which solutions to the above differential equation satisfy~~. so that y_g satisfies the above differential equation

$$p(p-1) - 2 = 0$$

$$p^2 - p - 2 = 0$$

$$(p-2)(p+1) = 0$$

$$p = 2, -1$$

(c) (5 points) Solve the above equation in p to determine two linearly independent solutions to the above differential equation.

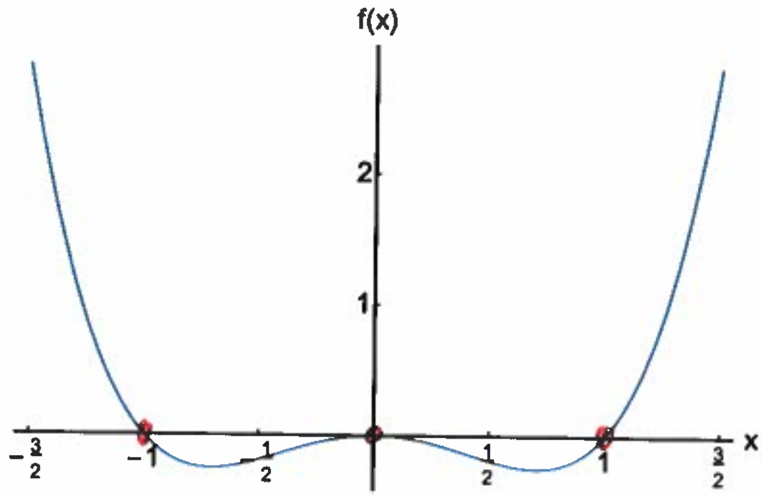
$$y = x^2$$

$$y = 1/x.$$

6. (10 points) Consider the following differential equation

$$\frac{dx}{dt} = f(x),$$

where $f(x)$ is plotted below.



(a) (5 points) **Short Answer:** On the figure, indicate the location of any fixed points, i.e. equilibrium points.

(b) (5 points) **Short Answer:** On one axis sketch the solutions curves for this differential equation as functions of time. Include enough solution curves so that they illustrate all possible qualitatively different possibilities.