

## Homework #1

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$$3x + y - 2z = 3$$

$$x - 8y - 14z = -14$$

$$x + 2y + z = 2$$

Solution:

The augmented matrix is given by:

$$\left[ \begin{array}{ccc|c} 3 & 1 & -2 & 3 \\ 1 & -8 & -14 & -14 \\ 1 & 2 & 1 & 2 \end{array} \right] \begin{array}{l} -3R2 \\ -R2 \end{array} \Rightarrow \left[ \begin{array}{ccc|c} 0 & 25 & 40 & 45 \\ 1 & -8 & -14 & -14 \\ 0 & 10 & 15 & 16 \end{array} \right] /5$$

$$\Rightarrow \left[ \begin{array}{ccc|c} 0 & 5 & 8 & 9 \\ 1 & -8 & -14 & -14 \\ 0 & 10 & 15 & 16 \end{array} \right] \begin{array}{l} \\ -2R1 \end{array} \Rightarrow \left[ \begin{array}{ccc|c} 0 & 5 & 8 & 9 \\ 1 & -8 & -14 & -14 \\ 0 & 0 & -1 & -2 \end{array} \right]$$

Therefore,

$$\boxed{z = 2}$$

$$\Rightarrow 5y + 16 = 9$$

$$\Rightarrow 5y = -7$$

$$\Rightarrow \boxed{y = -\frac{7}{5}}$$

$$\Rightarrow x - \frac{56}{5} + 28 = -14$$

$$\Rightarrow \boxed{x = \frac{14}{5}}$$

Ex

$$x_1 + x_2 - x_3 + 2x_4 = 1$$

$$x_1 + x_2 - x_3 - x_4 = -1$$

$$x_1 + 2x_2 + x_3 + 2x_4 = -1$$

$$2x_1 + 2x_2 + x_3 + x_4 = 2$$

Solution:

The augmented matrix is given by:

$$\left[ \begin{array}{cccc|c} 1 & 1 & -1 & 2 & 1 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & 2 & 1 & 2 & -1 \\ 2 & 2 & 1 & 1 & 2 \end{array} \right] \begin{array}{l} -R1 \\ -R1 \\ -R1 \\ -2R1 \end{array} \Rightarrow \left[ \begin{array}{cccc|c} 1 & 1 & -1 & 2 & 1 \\ 0 & 0 & 0 & -3 & -2 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 3 & -3 & 0 \end{array} \right]$$

Therefore,

$$x_4 = \frac{2}{3}$$

$$\Rightarrow 3x_3 - 3x_4 = 0$$

$$\Rightarrow x_3 = x_4 = \frac{2}{3}$$

$$\Rightarrow x_2 + 2x_3 = -2$$

$$x_2 + \frac{4}{3} = -2$$

$$\Rightarrow x_2 = -\frac{10}{3}$$

$$\Rightarrow x_1 - \frac{10}{3} - \frac{2}{3} + \frac{4}{3} = 1$$

$$\Rightarrow x_1 = \frac{11}{3}$$

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$$\begin{aligned}2x - 4y + 6z &= 2 \\ -3x + 6y - 9z &= 3\end{aligned}$$

Solution:

The augmented matrix is given by:

$$\begin{aligned}\begin{bmatrix} 2 & -4 & 6 & 2 \\ -3 & 6 & -9 & 3 \end{bmatrix} \begin{matrix} \frac{0}{2} \\ \frac{0}{3} \end{matrix} &\Rightarrow \begin{bmatrix} 1 & -2 & 3 & 1 \\ -1 & 2 & -3 & 1 \end{bmatrix} +R_1 \\ &\Rightarrow \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}\end{aligned}$$

Therefore, there is no solution since  $2 \neq 0$ .

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Determine conditions on  $a, b, c, d$  so that the following system of equations has a solution.

$$x_1 - x_2 + x_3 + x_4 = a$$

$$x_1 + x_2 - 2x_3 + 3x_4 = b$$

$$3x_1 - 2x_2 + 3x_3 - 2x_4 = c$$

$$2x_1 - 3x_3 + 2x_4 = d$$

Solution:

The augmented matrix is given by:

$$\begin{aligned}\begin{bmatrix} 1 & -1 & 1 & 1 & a \\ 1 & 1 & -2 & 3 & b \\ 3 & -2 & 3 & -2 & c \\ 0 & 2 & -3 & 2 & d \end{bmatrix} \begin{matrix} \\ -R_1 \\ -3R_1 \\ \end{matrix} &\Rightarrow \begin{bmatrix} 1 & -1 & 1 & 1 & a \\ 0 & 2 & -3 & 2 & b-a \\ 0 & 1 & 0 & -5 & c-3a \\ 0 & 2 & -3 & 2 & d \end{bmatrix} \begin{matrix} +R_3 \\ -2R_3 \\ \\ -R_2 \end{matrix}\end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & -4 & c-2a \\ 0 & 0 & -3 & 12 & b-a-2c+6a \\ 0 & 1 & 0 & -5 & c-3a \\ 0 & 0 & 0 & 0 & d-b+a \end{bmatrix}$$

Therefore, for a solution to exist!

$$\boxed{d-b+a=0.}$$

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Determine if the following homogeneous system has nontrivial solutions!

$$x+y+z=0$$

$$3x-y-2z=0$$

$$2x-2y-4z=0$$

$$x+3y+6z=0.$$

The augmented matrix is given by!

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 3 & -1 & -2 & 0 \\ 2 & -2 & -4 & 0 \\ 1 & 3 & 6 & 0 \end{bmatrix} \begin{matrix} \\ -3R_1 \\ -2R_1 \\ -R_1 \end{matrix} \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore,

$$y = -2z$$

$$x - 2z + 2z = 0$$

$$\Rightarrow x = 0.$$

The solution set is given by

$$\boxed{\begin{matrix} x=0 \\ y=-2z \end{matrix}}$$