Homework #10

#4.

$$2y'' + 3y' + y = \cos(x)$$

$$y_H = e^{\lambda x}$$

$$\Rightarrow 2\lambda^2 + 3\lambda + 1 = 0$$

$$\Rightarrow (2\lambda+1)(\lambda+1) = 0$$

$$\Rightarrow \lambda = -\tfrac{1}{2},\ \lambda = -1.$$

Therefore, $y_H = c_1 e^{-\frac{1}{2}x} + c_2 e^{-x}$. For the particular solution guess

$$y_p = A\cos(x) + B\sin(x)$$

$$\Rightarrow \big(-2A\cos(x) - 2B\sin(x) - 3A\sin(x) + 3B\cos(x) + A\cos(x) + B\sin(x)\big) = \cos(x)$$

$$\Rightarrow \begin{matrix} -A + 3B = 1 \\ -B - 3A = 0 \end{matrix} \Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 3 & 1 & 0 \end{bmatrix} \begin{matrix} \\ -3R1 \end{matrix} \Rightarrow \left[\begin{array}{cc|c} 1 & -3 & -1 \\ 0 & 10 & 3 \end{array}\right]$$

$$\Rightarrow B = \tfrac{1}{6},\ A - \tfrac{1}{2} = -1 \Rightarrow A = -\tfrac{1}{2}$$

Therefore,

$$y = c_1 e^{-\frac{1}{2}x} + c_2 e^{-x} - \tfrac{1}{2}\cos(x) + \tfrac{1}{6}\sin(x).$$

#9.

$$4y'' + 16y = 3\cos(2x)$$

$$y_H = c_1\cos(2x) + c_2\sin(2x)$$

Since $3\cos(2x) \in \operatorname{span}\{\cos(2x), \sin(2x)\}$ we make the guess:

$$y_p = Ax\cos(2x) + Bx\sin(2x)$$

$$y_p' = A\cos(2x) - 2Ax\sin(2x) + B\sin(2x) + 2Bx\cos(2x)$$

$$y_p'' = -4A\sin(2x) - 4Ax\cos(2x) + 4B\cos(2x) - 4Bx\sin(2x)$$

$$\Rightarrow -16A\sin(2x) + 16B\cos(2x) = 3\cos(2x).$$

Therefore, the general solution is given by:

$$y = c_1\cos(2x) + c_2\sin(2x) + \frac{3}{16}x\sin(2x)$$

#11.

$$y'' + 8y' = 2x^2 - 7x + 3$$

$$y_H = c_1 + c_2e^{-8x}$$

We make the guess

$$y_p = Ax^3 + Bx^2 + Cx$$

$$y_p' = 3Ax^2 + 2Bx + C$$

$$y_p'' = 6Ax + 2B$$

$$\Rightarrow 6Ax + 2B + 24Ax^2 + 16Bx + 8C = 2x^2 - 7x + 3$$

$$24A = 2$$

$$\Rightarrow A = 1/12$$

$$6(1/12) + 16B = -7$$

$$1/2 + 16B = -7$$

$$B = -15/32$$

$$8C + -15/16 = 3$$

$$8C = 63/16$$

$$C = 63/(16\cdot 8)$$

$$\Rightarrow y = c_1 + c_2e^{-8x} + 1/12\,x^3 - 15/32\,x^2 + 63/(16\cdot 8)\,x$$

#18.

$$y'' + 4y' + 4y = 2x - \sin(3x)$$

$$y_H = e^{\lambda x}$$

$$\Rightarrow \lambda^2 + 4\lambda + 4 = 0$$

$$\Rightarrow (\lambda + 2)^2 = 0$$

$$\Rightarrow y_H = c_1 e^{-2x} + c_2 x e^{-2x}$$

$$y_p = Ax + B + C\sin(3x) + D\cos(3x)$$

$$y_p' = A + 3C\cos(3x) - 3D\sin(3x)$$

$$y_p'' = -9C\sin(3x) - 9D\cos(3x)$$

$$\Rightarrow -9C\sin(3x) - 9D\cos(3x) + 4A + 12C\cos(3x) - 12D\sin(3x)$$
$$+ 4Ax + 4B + 4C\sin(3x) + 4D\cos(3x) = 2x - \sin(3x)$$

$$\Rightarrow -5C - 12D = -1$$
$$-5D + 12C = 0$$
$$4A + 4B = 0$$
$$4Ax = 2x$$

$$\Rightarrow A = 1/2$$
$$B = -1/2$$
$$D = 12/5\, C$$
$$-5C - 144/5\, C = -1$$
$$-25C - 144C = -5$$
$$C = \frac{-20}{144} = \frac{-5}{36} \quad C = 5/169$$

$$\Rightarrow y = c_1 e^{2x} + c_2 x e^{2x} + 1/2\, x - 1/2 - 5/36 \cos(3x) - 1/3 \sin(3x).$$

$$y = \frac{145}{228} e^{-2x} + \frac{19}{71} x e^{-2x} + \frac{1}{2}x - \frac{1}{2} + \frac{12}{169}\cos(3x) + \frac{5}{169}\sin(3x)$$

#23.

$$y_p = Ax^2 + Bx + C + Dx\cos(x) + Ex\sin(x).$$

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$$A + Bx + Ce^{4x} + D\sin(4x) + E\cos(4x)$$
$$+ Fx^2\cos(2x) + Gx^2\sin(2x) + Hx^3\cos(2x) + Ix^3\sin(2x)$$