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#9.

Not linear since

$$T(0) = 1 \neq 0.$$

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#18.

$$T = \begin{bmatrix} 1 & -1 & 3 & -1 \\ 2 & 3 & -1 & -2 \\ 3 & 7 & -5 & -3 \end{bmatrix}$$

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#22

$$c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & a \\ 1 & 1 & 1 & b \\ 0 & 1 & 1 & c \end{array} \right] \xrightarrow{-R_1} \left[\begin{array}{ccc|c} 1 & -1 & 0 & a \\ 0 & 2 & 1 & b-a \\ 0 & 1 & 1 & c \end{array} \right] \xrightarrow{\cdot 1/2} \left[\begin{array}{ccc|c} 1 & -1 & 0 & a \\ 0 & 1 & 1/2 & (a-b)/2 \\ 0 & 1 & 1 & c \end{array} \right] \xrightarrow{-R_2}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & b/2 \\ 0 & 1 & -1/2 & a-b/2 \\ 0 & 0 & 3/2 & (c-2a+b)/2 \end{array} \right] \xrightarrow{\cdot 2/3} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & b/2 \\ 0 & 1 & -1/2 & a-b/2 \\ 0 & 0 & 1 & (2c-4a+2b)/3 \end{array} \right] \xrightarrow{-1/2 R_3, +1/2 R_3}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & (-2c+4a+b)/6 \\ 0 & 1 & 0 & (2a-b+2c)/6 \\ 0 & 0 & 1 & (2c-4a+2b)/3 \end{array} \right] \Rightarrow T \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 & -1/3 \\ 1/3 & -1/3 & 1/3 \\ -1/3 & 2/3 & 2/3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\Rightarrow T = \begin{bmatrix} 2/3 & 1/3 & -1/3 \\ 1/3 & -1/3 & 1/3 \\ -1/3 & 2/3 & 2/3 \end{bmatrix}, T \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 4/3 + 1/3 + 4/3 \\ 2/3 - 1/3 - 4/3 \\ -2/3 + 2/3 - 8/3 \end{bmatrix}$$

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$$\begin{aligned}(S-4T)\begin{bmatrix} x \\ y \end{bmatrix} &= S\begin{bmatrix} x \\ y \end{bmatrix} - 4T\begin{bmatrix} x \\ y \end{bmatrix} \\ &= \begin{bmatrix} 2x-y \\ x+2y \end{bmatrix} - 4\begin{bmatrix} x+3y \\ x-y \end{bmatrix} \\ &= \begin{bmatrix} -2x-13y \\ -3x+6y \end{bmatrix}\end{aligned}$$

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$$\begin{aligned}(TS)(ax+bx) &= T(ax-2a+bx) \\ &= ax+2a-2a+bx \\ &= ax+bx.\end{aligned}$$