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$$T(x_1, x_2) = \begin{bmatrix} 5x_1 + 3x_2 \\ -6x_1 - 4x_2 \end{bmatrix}$$

(a) $[T]_\alpha^\alpha = \begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}$

(b). $[I]_\alpha^\beta = \left[[e_1]_\beta \,\middle|\, [e_2]_\beta \right]$

$$= \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix}_\beta \,\middle|\, \begin{bmatrix} 0 \\ 1 \end{bmatrix}_\beta \right]$$

I am going to find $[I]_\alpha^\beta$ and invert.

$$[I]_\beta^\alpha = \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow [I]_\alpha^\beta = \left([I]_\beta^\alpha\right)^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -1 & -1 \end{bmatrix}$$

(c) $[I]_\beta^\alpha = \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}$

d) $[T]_\alpha^\alpha \vec{v}_\alpha = \vec{w}_\alpha$

$$\Rightarrow [T]_\alpha^\alpha [I]_\beta^\alpha \vec{v}_\beta = [I]_\beta^\alpha \vec{w}_\beta$$

$$\Rightarrow \left([I]_\beta^\alpha\right)^{-1} [T]_\alpha^\alpha [I]_\beta^\alpha \vec{v}_\beta = \vec{w}_\beta$$

$$\Rightarrow [T]_\beta^\beta = \left([I]_\beta^\alpha\right)^{-1} [T]_\alpha^\alpha [I]_\beta^\alpha$$

$$= \begin{bmatrix} -1 & -2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ -2 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -9 \\ 0 & -1 \end{bmatrix}$$

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$D: P_2 \to P_2$ defined by

$D(ax^2+bx+c) = 2ax+b.$

(a) $D(1) = 0 \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$D(x) = 1 \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$D(x^2) = 2x \Rightarrow \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

$\Rightarrow [T]_\alpha^\alpha = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $D(x^2+1) = 2x \Rightarrow \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

$D(x+1) = 1 \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$D(2x^2+1) = 4x \Rightarrow \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$

Now $[I]_\beta^\alpha = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, $[I]_\alpha^\beta = \left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_\beta \middle| \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_\beta \middle| \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_\beta \right] = ([I]_\beta^\alpha)^{-1}$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \\ -R_1 \end{array} \to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} -R2 \\ \\ +R2 \end{array} \to \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} -R3 \\ \\ \\ \end{array}$

$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -2 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \Rightarrow [I]_\alpha^\beta = \begin{bmatrix} 2 & -2 & -1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$

(c). $[I]_\beta^\alpha = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

(d). $[T]_\beta^\beta = \begin{bmatrix} 2 & -2 & -1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & -1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} -4 & 2 & -8 \\ 2 & 0 & 4 \\ 2 & -1 & 4 \end{bmatrix}$

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$$T(x_1, x_2) = \begin{bmatrix} 5x_1 + 3x_2 \\ -6x_1 - 4x_2 \end{bmatrix}$$

a) $[T]_\alpha^\alpha = \boxed{\begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}}$

b) $[I]_\alpha^\beta = \left[[e_1]_\beta \,\middle|\, [e_2]_\beta \right]$

$$= \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix}_\beta \,\middle|\, \begin{bmatrix} 0 \\ 1 \end{bmatrix}_\beta \right]$$

$$[I]_\beta^\alpha = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow [I]_\alpha^\beta = \left([I]_\beta^\alpha \right)^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}}$$

c) $[I]_\beta^\alpha = \boxed{\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}}$

d) $[T]_\alpha^\alpha \vec{V}_\alpha = \vec{W}_\alpha \Rightarrow [T]_\alpha^\alpha [I]_\beta^\alpha \vec{V}_\beta = [I]_\beta^\alpha \vec{W}_\beta$

$$\Rightarrow \left([I]_\beta^\alpha \right)^{-1} [T]_\alpha^\alpha \left([I]_\beta^\alpha \right) \vec{V}_\beta = \vec{W}_\beta$$

$$\Rightarrow [T]_\beta^\beta = \left([I]_\beta^\alpha \right)^{-1} [T]_\alpha^\alpha [I]_\beta^\alpha$$

$$= \begin{bmatrix} 1 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 29 & 18 \\ -45 & -28 \end{bmatrix}}$$

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a. $\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} -1 & -2 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$

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$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\boxed{\lambda = \frac{5 \pm \sqrt{33}}{2}}$

$\lambda = \frac{5-\sqrt{33}}{2} \Rightarrow \boxed{\begin{bmatrix} \frac{-3+\sqrt{33}}{6} \\ 1 \end{bmatrix}}$

$\lambda = \frac{5+\sqrt{33}}{2} \Rightarrow \boxed{\begin{bmatrix} \frac{-3-\sqrt{33}}{6} \\ 1 \end{bmatrix}}$

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$\begin{bmatrix} 3 & 1 & -1 \\ 0 & 0 & -2 \\ 0 & 1 & 2 \end{bmatrix}$

$\lambda = \boxed{3, 1 \pm i}$

$\boxed{E_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad E_{1+i} \begin{bmatrix} 1 \\ -1+i \\ 1 \end{bmatrix} \quad E_{1-i} \begin{bmatrix} 1 \\ -1-i \\ 1 \end{bmatrix}}$