Homework #2: Solutions

#14.

$$B=\begin{bmatrix} 2 & -1 \\ -3 & -2 \\ 0 & 4 \end{bmatrix}, \quad C=\begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix}, \quad D=\begin{bmatrix} 0 & 1 \\ 3 & -1 \end{bmatrix}$$

$$\Rightarrow C+D=\begin{bmatrix} 2 & 0 \\ 4 & 4 \end{bmatrix}$$

$$\Rightarrow B(C+D)=\begin{bmatrix} 2 & -1 \\ -3 & -2 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 4 \end{bmatrix}=\begin{bmatrix} 0 & -4 \\ -14 & -8 \\ 16 & 16 \end{bmatrix}$$

#18.

$$A=\begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 2 & -1 \end{bmatrix} \Rightarrow A^2 \text{ is undefined.}$$

Since A^2 is undefined it follows that $A^3 = A \cdot A^2$ is undefined as well.

#20.

$$x_1 - 3x_2 + x_3 - 5x_4 = 2$$
$$x_1 + x_2 - x_3 + x_4 = 1$$
$$x_1 - x_2 - x_3 + 6x_4 = 6.$$

Letting $A=\begin{bmatrix} 1 & -3 & 1 & -5 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 6 \end{bmatrix}$, it follows that:

$$A \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}.$$

#4.

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & -2 \\ 1 & 1 & 5 \end{bmatrix}.$$

The augmented matrix is given by:

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 3 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 & 1 & 0 \\ 1 & 1 & 5 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 0 & 1 & 0 \\ 2 & -1 & 3 & 1 & 0 & 0 \\ 1 & 1 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -2R1 \\ -R1 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 0 & 1 & 0 \\ 0 & -3 & 7 & 1 & -2 & 0 \\ 0 & 0 & 7 & 0 & -1 & 1 \end{array}\right] \begin{array}{l} \\ \div(-3) \\ \div(7) \end{array} \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 0 & 1 & 0 \\ 0 & 1 & -7/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 1 & 0 & -1/7 & 1/7 \end{array}\right] \begin{array}{l} -R2 \\ \\ \\ \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1 & -7/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 1 & 0 & -1/7 & 1/7 \end{array}\right] \begin{array}{l} -1/3 R3 \\ +7/3 R3 \\ \\ \end{array} \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 10/21 & -1/21 \\ 0 & 1 & 0 & -1/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 0 & -1/7 & 1/7 \end{array}\right].$$

Therefore,

$$A^{-1} = \begin{bmatrix} 1/3 & 10/21 & -1/21 \\ -1/3 & 1/3 & 1/3 \\ 0 & -1/7 & 1/7 \end{bmatrix}.$$

#6

$$A = \begin{bmatrix} 0 & -1 & 3 \\ 0 & -4 & 1 \\ 2 & -1 & 3 \end{bmatrix}$$

The augmented matrix is given by:

$$\left[\begin{array}{ccc:ccc} 0 & -1 & 3 & 1 & 0 & 0 \\ 0 & -4 & 1 & 0 & 1 & 0 \\ 2 & -1 & 3 & 0 & 0 & 1 \end{array}\right] \Rightarrow \begin{array}{c} \left[\begin{array}{ccc:ccc} 2 & -1 & 3 & 0 & 0 & 1 \\ 0 & -4 & 1 & 0 & 1 & 0 \\ 0 & -1 & 3 & 1 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} \div 2 \\ \\ \\ \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & -\frac{1}{2} & \frac{3}{2} & 0 & 0 & \frac{1}{2} \\ 0 & -4 & 1 & 0 & 1 & 0 \\ 0 & -1 & 3 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \div -4 \\ \\ \end{array} \Rightarrow \left[\begin{array}{ccc:ccc} 1 & -\frac{1}{2} & \frac{3}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 \\ 0 & -1 & 3 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} +\frac{1}{2}R2 \\ \\ +R2 \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{11}{8} & 0 & -\frac{1}{8} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 \\ 0 & 0 & \frac{11}{4} & 1 & -\frac{1}{4} & 0 \end{array}\right] \begin{array}{l} \\ \\ \times \frac{4}{11} \end{array} \Rightarrow \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{11}{8} & 0 & -\frac{1}{8} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{4}{11} & -\frac{1}{11} & 0 \end{array}\right] \begin{array}{l} -\frac{11}{8}R_3 \\ +\frac{1}{4}R_3 \\ \\ \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{11} & -\frac{5}{44} & 0 \\ 0 & 0 & 1 & \frac{4}{11} & -\frac{1}{11} & 0 \end{array}\right].$$

Therefore,

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{11} & -\frac{5}{44} & 0 \\ \frac{4}{11} & -\frac{1}{11} & 0 \end{bmatrix}$$