Homework #3

pg. 42, #6

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \times -1 \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] +3R3$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] -2R2 \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Therefore, $A^{-1} = \begin{bmatrix} 1 & 2 & -6 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.

pg. 50, #8

$$\det\left(\begin{bmatrix} 2 & -1 & 5 & 6 \\ 0 & 3 & 4 & 0 \\ 0 & 1 & 5 & 2 \\ 0 & 1 & -3 & 0 \end{bmatrix}\right) = 2\det\left(\begin{bmatrix} 3 & 4 & 0 \\ 1 & 5 & 2 \\ 1 & -3 & 0 \end{bmatrix}\right) = -4\det\left(\begin{bmatrix} 3 & 4 \\ 1 & -3 \end{bmatrix}\right)$$

$$\Rightarrow \det(A) = -4(-13)$$
$$= 52.$$

pg. 50, #12.

$$\det\left(\begin{bmatrix} 2 & -1 & 3 & 1 \\ -1 & 2 & -1 & 4 \\ 1 & -1 & 3 & 1 \\ 3 & 2 & -1 & 5 \end{bmatrix}\right)\begin{matrix} -2R3 \\ +R3 \\ \\ -3R3 \end{matrix} = \det\left(\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 1 & 2 & 5 \\ 1 & -1 & 3 & 1 \\ 0 & 5 & -10 & 2 \end{bmatrix}\right)$$

$$= \det\left(\begin{bmatrix} 1 & -3 & -1 \\ 1 & 2 & 5 \\ 5 & -10 & 2 \end{bmatrix}\right)\begin{matrix} \\ -R1 \\ -5R1 \end{matrix} \Rightarrow \det\left(\begin{bmatrix} 1 & -3 & -1 \\ 0 & 5 & 6 \\ 0 & 5 & 7 \end{bmatrix}\right)$$

$$= \det\left(\begin{bmatrix} 5 & 6 \\ 5 & 7 \end{bmatrix}\right) = 5.$$

pg. 57, #4

$$\det\left(\begin{bmatrix} 2 & -1 & -3 \\ 1 & 1 & 3 \\ 6 & 0 & 0 \end{bmatrix}\right) = 6\det\left(\begin{bmatrix} -1 & -3 \\ 1 & 3 \end{bmatrix}\right) = 6(-3+3) = 0.$$

Therefore, this matrix is not invertible.

pg. 57, #12.

$$e^{t}x + e^{2t}y + e^{-t}z = 1$$
$$e^{t}x + 2e^{2t}y - e^{-t}z = t$$
$$e^{t}x + 4e^{2t} + e^{-t}z = t^2$$

$$A = \begin{bmatrix} e^{t} & e^{2t} & e^{-t} \\ e^{t} & 2e^{2t} & -e^{-t} \\ e^{t} & 4e^{2t} & e^{-t} \end{bmatrix} \Rightarrow \det(A) = e^{t}\det\left(\begin{bmatrix} 1 & e^{t} & e^{-2t} \\ 1 & 2e^{t} & -e^{-2t} \\ 1 & 4e^{t} & e^{-2t} \end{bmatrix}\right)$$

$$\Rightarrow \det(A) = e^{-t}\left(\begin{bmatrix} 1 & e^{t} & e^{-2t} \\ 0 & e^{t} & -2e^{-2t} \\ 0 & 3e^{t} & 0 \end{bmatrix}\right)$$

$$= e^{-t}\cdot 6e^{-t}$$

$$= 6e^{-2t}.$$

$$A_1 = \begin{bmatrix} 1 & e^{2t} & e^{-t} \\ t & 2e^{2t} & -e^{-t} \\ t^2 & 4e^{2t} & e^{-t} \end{bmatrix} \begin{matrix} \\ -tR1 \\ -t^2R1 \end{matrix}$$

$$\Rightarrow \det(A_1) = \det\left(\begin{bmatrix} 1 & e^{2t} & e^{-t} \\ 0 & e^{2t}(2-t) & -e^{-t}(1+t) \\ 0 & e^{2t}(4-t^2) & e^{-t}(1-t^2) \end{bmatrix}\right)$$

$$= e^{t}(2-t)(1-t^2) + e^{t}(4-t^2)(1+t)$$

$$= e^{t}(2-t)(1+t)(1-t+2+t)$$

$$= 3e^{t}(2-t)(1+t).$$

$$\Rightarrow x = \frac{3e^{t}(2-t)(1+t)}{6e^{-2t}}$$

$$= \frac{e^{3t}(2-t)(1+t)}{3}.$$

Be nice on grading this problem :)