pg. 74, #5.

$(x_1,y_1)+(x_2,y_2)=(x_1x_2,y_1y_2)$

$c(x,y)=(x^c,y^c)$

Solution:

1. $(x_1,y_1)+(x_2,y_2)=(x_1x_2,y_1y_2)$
$=(x_2x_1,y_2y_1)$
$=(x_2,y_2)+(x_1,y_2)$

2. $(x_1,y_1)+((x_2,y_2)+(x_3,y_3))=(x_1,y_1)+(x_2x_3,y_2y_3)$
$=(x_1x_2x_3,y_1y_2y_3)$
$=(x_1x_2,y_1y_2)+(x_3,y_3)$
$=((x_1,y_1)+(x_2,y_2))+(x_3,y_3)$

3. $(x,y)+(1,1)=(x\cdot 1,y\cdot 1)=(x,y)$.
Therefore, $0=(1,1)$.

4. $(x_1,y_1)+(x_1^{-1},y_1^{-1})=(x_1x_1^{-1},y_1y_1^{-1})=(1,1)$.

5. $c((x_1,y_1)+(x_2,y_2))=c(x_1y_1,x_2y_2)$
$=(x_1^cy_1^c,x_2^cy_2^c)$
$=(x_1^c,y_1^c)+(x_2^c,y_2^c)$
$=c(x_1,y_1)+c(x_2,y_2)$.

6. $(c+d)(x_1,y_1)=(x_1^{c+d},y_1^{c+d})$
$=(x_1^cx_1^d,y_1^cy_1^d)$
$=(x_1^c,y_1^c)+(x_1^d,y_1^d)$
$=c(x_1,y_1)+d(x_1,y_1)$

7. $c(d(x_1,y_1))=c(x_1^d,y_1^d)$
$=(x_1^{dc},y_1^{dc})$
$=d(x_1^c,y_1^c)$
$=d(c(x_1,y_1))$.

8. $1\cdot(x_1,y_1)=(x_1^1,y_1^1)=(x_1,y_1)$. ■

#4

b) ~~Not a subspace~~ Subspace

c) ~~Not a subspace~~ Subspace

d) ~~Subspace~~ not a subspace

e) ~~Subspace~~ not a subspace

#5

No, if $\vec{x}_1, \vec{x}_2$ solve $A\vec{x}=b$ then $A(\vec{x}_1+\vec{x}_2)=A\vec{x}_1+A\vec{x}_2=2\vec{b}$.

#12.

Is $\begin{bmatrix} 3 & 5 \\ 3 & 4 \end{bmatrix}$ in span$\left\{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right\}$?

$$\Rightarrow \begin{bmatrix} 3 & 5 \\ 3 & 4 \end{bmatrix} = c_1\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + c_2\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + c_3\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 3 &= c_1+c_2+c_3 \\ 5 &= c_1+c_2 \\ 3 &= c_2 \\ 4 &= c_1-c_3 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 1 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & -1 & 4 \end{array}\right] \begin{array}{l} \\ \Rightarrow -R1 \\ \\ \Rightarrow -R1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & -1 & -2 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ +R3 \end{array}$$

$+1 \quad -2 \qquad 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & -2 & 4 \end{array}\right]$$

$c_2 = 3$

$c_3 = -2$

$c_1 = 2$

This system is inconsistent so

$$\begin{bmatrix} 3 & 5 \\ 3 & 4 \end{bmatrix} \notin \text{span}\left\{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right\}$$

#20

Determine if $x^3+x^2$, $x^2+x$, $x+1$ span $P_3$. If $ax^3+bx^2+cx+d$ lies in span$\{P_1, P_2, P_3\}$ it follows that

$$ax^3+bx^2+cx+d = c_1(x^3+x^2)+c_2(x^2+x)+c_3(x+1)$$

$$\rightarrow \begin{matrix} c_1 = a \\ c_1+c_2 = b \\ c_2+c_3 = c \\ c_3 = d \end{matrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 1 & 1 & 0 & b \\ 0 & 1 & 1 & c \\ 0 & 0 & 1 & d \end{array}\right] \begin{matrix} \\ -R1 \\ \\ \\ \end{matrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b-a \\ 0 & 1 & 1 & c \\ 0 & 0 & 1 & d \end{array}\right] \begin{matrix} \\ \\ -R2 \\ \\ \end{matrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b-a \\ 0 & 0 & 1 & c-b+a \\ 0 & 0 & 1 & d \end{array}\right]$$

This equation is inconsistent so $P_1, P_2, P_3$ do not span cubic polynomials.