~~pg. 188, #10~~

$x^2y''+xy'-y=0$

$y(1)=0$

$y'(1)=-1$

$\frac{d}{dx}x^{-1}=-1x^{-2}$, $\frac{d}{dx}x=1$

$\frac{d^2}{dx^2}x^{-1}=2x^{-3}$, $\frac{d^2}{dx^2}x=0$

$x^2\frac{d^2}{dx^2}(x^{-1})+x\frac{d}{dx}(x^{-1})-x^{-1}=\frac{2}{x}-\frac{1}{x}-\frac{1}{x}=0$

$x^2\frac{d^2}{dx^2}(x)+x\frac{d}{dx}(x)-x^{-1}=0$

Therefore,

$y=\frac{c_1}{x}+c_2x$

$\Rightarrow y(1)=c_1+c_2=0$

$y'(x)=-\frac{c_1}{x^2}+c_2$

$\Rightarrow y'(1)=-c_1+c_2=-1$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & 1 & -1 \end{array}\right] +R1 \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 2 & -1 \end{array}\right]$

$\Rightarrow c_2=-1/2$

$c_1=1/2$

Therefore,

$y=1/2\cdot 1/x-1/2\,x.$

#16

$y_p = x^4/15$

$y_p' = 4x^3/15$

$y_p'' = 4x^2/5$

$y_p''' = 8x^1/5$

$\Rightarrow x^3y_p''' + x^2y_p'' - 2xy_p' + 2y = \frac{8x^4}{5} + \frac{4x^4}{5} - \frac{8x^4}{15} + \frac{2x^4}{15}$

$= \frac{12x^4}{5} - \frac{6x^4}{15}$

$= \frac{12x^4}{5} - \frac{2x^4}{5} = 2x^4.$

$y = c_1x + c_2x^2 + c_3x^{-1} + x^4/15$

$y(-1) = -c_1 + c_2 - c_3 + 1/15$

$y'(x) = c_1 + 2c_2x - c_3x^{-2} + 4/15x^3$

$y'(-1) = c_1 - 2c_2 + c_3 - 4/15$

$y''(x) = 2c_2 + 2c_3x^{-3} + 4x^2/5$

$y''(-1) = 2c_2 - 2c_3 + 4/5$

$\rightarrow \left[\begin{array}{ccc|c} -1 & 1 & -1 & -1/15 \\ 1 & -2 & 1 & 4/15 \\ 0 & 2 & -2 & -4/5 \end{array}\right]$ $\leftarrow$ I am fine with them having the correct matrix.

pg. 201

#4

$2y'' - 5y' - y = 0$

$y = e^{\lambda x}$

$\Rightarrow 2\lambda^2 e^{\lambda x} - 5\lambda e^{\lambda x} - e^{\lambda x} = 0$

$\Rightarrow 2\lambda^2 - 5\lambda - 1 = 0$

$$\Rightarrow \lambda = \frac{5 \pm \sqrt{25+8}}{4}$$

$$= \frac{5 \pm \sqrt{33}}{4}$$

$$\Rightarrow y = c_1 \exp\left(\left(\frac{5 \pm \sqrt{33}}{4}\right)x\right) + c_2 \exp\left(\left(\frac{5 - \sqrt{33}}{4}\right)x\right)$$

Nevermind on #8, to many parts for this problem to be fair.

#8

$y''' - 5y'' + 8y' - 4y = 0$

$y = e^{\lambda x}$

$\Rightarrow \lambda^3 e^{\lambda x} - 5\lambda^2 e^{\lambda x} + 8\lambda e^{\lambda x} - 4e^{\lambda x} = 0$

$\Rightarrow \lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$

$\lambda = 1$ is cleary a root.

$$\begin{array}{r l}
 & \lambda^2 - 4\lambda + 4 \\
\lambda - 1 \,\big) & \lambda^3 - 5\lambda^2 + 8\lambda - 4 \\
 & -\lambda^3 - \lambda^2 \\
 & -4\lambda^2 + 8\lambda \\
 & -4\lambda^2 + 4\lambda \\
 & 4\lambda - 4 \\
 & 4\lambda - 4 \\
 & 0
\end{array}$$

Therefore,

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = (\lambda-1)(\lambda^2 - 4\lambda + 4)$$
$$= (\lambda - 1)(\lambda - 2)^2$$

We have a repeated root of $\lambda = 2$. Therefore assume another guess of the form:

$$y = ve^{2x}$$
$$y' = v'e^{2x} + 2ve^{2x}$$
$$y'' =$$

#12.

$$2y'' - 8y' + 14y = 0$$
$$\Rightarrow y'' - 4y' + 7y = 0$$
$$y = e^{\lambda x}$$
$$\Rightarrow \lambda^2 e^{\lambda x} - 4\lambda e^{\lambda x} + 7e^{\lambda x} = 0$$
$$\Rightarrow \lambda^2 - 4\lambda + 7 = 0$$
$$\Rightarrow \lambda = \frac{8 \pm \sqrt{16 - 28}}{2}$$
$$= 2 \pm \sqrt{4 - 7}$$
$$= 2 \pm \sqrt{3}\,i$$

Therefore,

$$y(x) = c_1 e^{2x} \cos(\sqrt{3}x) + c_2 e^{2x} \sin(\sqrt{3}x).$$

#18

$y'' + 4y' + 4y = 0$

$y(0) = 0$

$y'(0) = -1$

$y = e^{\lambda x}$

$\Rightarrow \lambda^2 e^{\lambda x} + 4\lambda e^{\lambda x} + 4e^{\lambda x} = 0$

$\Rightarrow \lambda^2 + 4\lambda + 4 = 0$

$\Rightarrow (\lambda + 2)^2 = 0$

$\Rightarrow \lambda = -2$

To obtain the other solution let

$y = ve^{-2x}$

$y' = v'e^{-2x} - 2ve^{-2x}$

$y'' = v''e^{-2x} - 4v'e^{-2x} - 4ve^{-2x}$

$\Rightarrow v'' - \cancel{4v'} - \cancel{4v} + \cancel{4v'} - 8v + \cancel{4v} = 0$

$\Rightarrow v'' = 0$

$\Rightarrow v = x$

Therefore,

$y = c_1 e^{-2x} + c_2 x e^{-2x}$

$y(0) = 0 \Rightarrow c_1 = 0$

$y'(x) = c_2 e^{-2x} - 2c_2 x e^{-2x}$

$y'(0) = c_2 = -1$

$\Rightarrow y(x) = -xe^{-2x}.$