

Section 3.1: Introduction to Differential Equations

Newton's Law of Cooling:



hot coal in freezer.

$T(t)$ temperature of coal

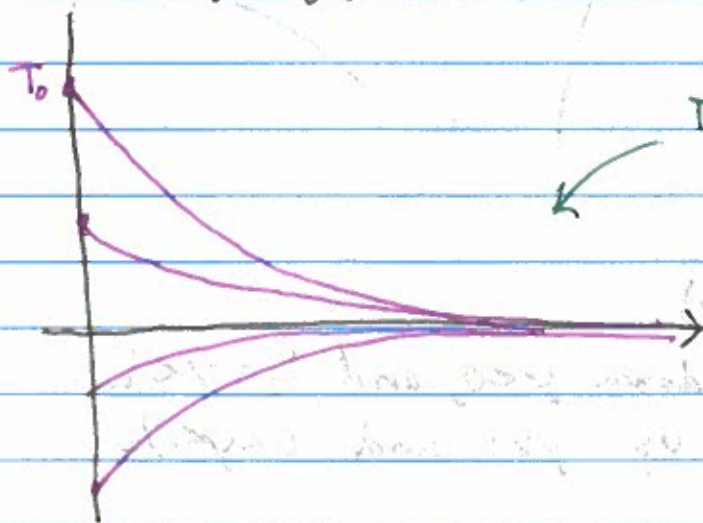
$$\frac{dT}{dt} = -KT, \quad T(0) = T_0$$

\uparrow Constant.

temperature is proportional to current temperature change.

Find $T(t)$

$$\Rightarrow T(t) = T_0 e^{-kt}$$



Temperature
equilibrates to
zero.

Example:

Solve

$$\frac{dy}{dx} = x^2$$

$$\Rightarrow y = \frac{1}{3}x^3 + C$$

general solution

Example:

Solve

$$\frac{dy}{dx} = x^3$$

$$y(1) = 1$$

$$\Rightarrow y(x) = \frac{1}{4}x^4 + C$$

$$y(1) = \frac{1}{4} + C = 1$$

$$\Rightarrow C = \frac{3}{4}$$

} Initial
value problem.

Example:

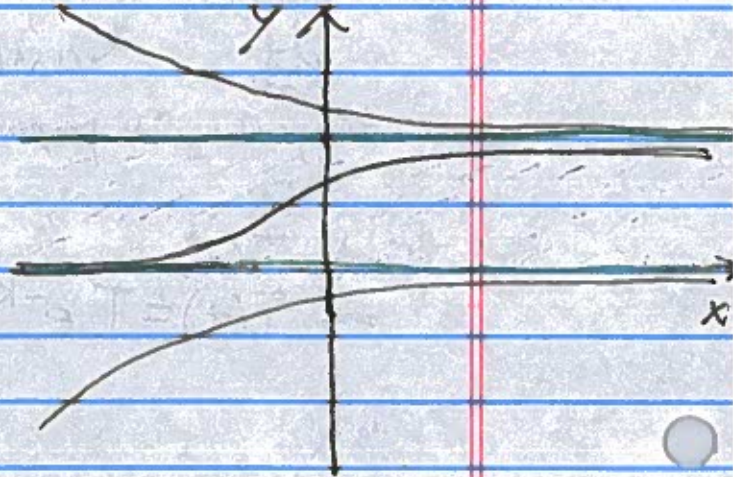
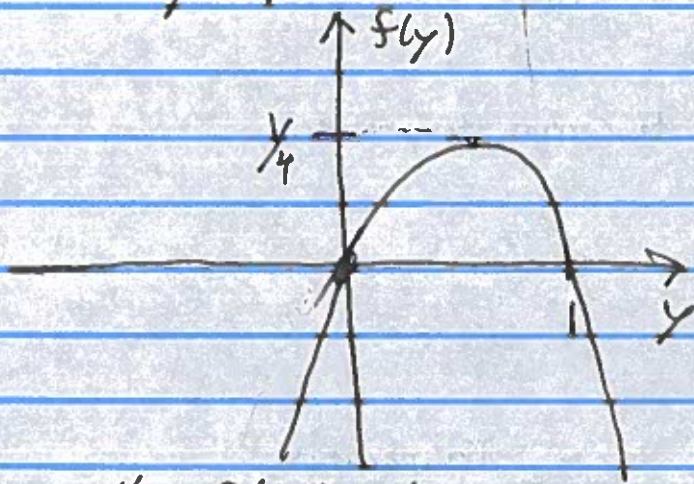
$$y' = y(1-y) = f(y)$$

What do the solution curves look like??

Exact solutions:

$$y = 0$$

$$y = 1$$



$$\begin{aligned} y'' &= f'(y) y' \\ &= f'(y) f(y) \\ &= (1-2y) y(1-y) \end{aligned}$$

Solution is concave down $y < 0$, and $\frac{1}{2} < y < 1$.
concave up $y > 1$ and $0 < y < \frac{1}{2}$