

Section 3.2: Separable Differential Equations.

Example:

Solve

$$\frac{dy}{dx} = ay, \quad y(0) = y_0$$

$$\Rightarrow \int_{y_0}^{y(x)} \frac{1}{y} dy = \int_0^x a dt$$

$$\ln(|y(x)|) - \ln(|y_0|) = ax$$

$$\Rightarrow \ln\left(\frac{|y(x)|}{|y_0|}\right) = ax$$

$$\Rightarrow y(x) = y_0 e^{ax}$$

Check:

$$\frac{dy}{dx} = y_0 a e^{ax} = a(y_0 e^{ax}) = ay(x)$$

$$y(0) = y_0 e^{a \cdot 0} = y_0$$

Example:

$$\frac{dy}{dx} = xy - 4x, \quad y(0) = y_0$$

$$\Rightarrow \frac{dy}{dx} = x(y-4), \quad y(0) = y_0$$

$$\Rightarrow \int_{y_0}^{y(x)} \frac{1}{y-4} dy = \int_0^x x dx$$

$$\Rightarrow \ln(|y(x)-4|) - \ln(|y_0-4|) = \frac{x^2}{2}$$

$$\Rightarrow \ln\left(\frac{|y(x)-4|}{|y_0-4|}\right) = \frac{x^2}{2}$$

$$\Rightarrow |y(x) - 4| = |y_0 - 4| e^{x^{3/2}}$$

Example:

$$\frac{dy}{dx} = y(\cos(x) - xy), y(0) = 1$$

$$\Rightarrow \int_1^y \frac{1}{y} dy = \int_0^x (\cos(x) - x) dx$$

$$\Rightarrow \ln(|y|) = \sin(x) - \sin(0) - \frac{x^2}{2}$$

$$\Rightarrow y(x) = \exp\left(\sin(x) - \frac{x^2}{2}\right)$$