

## Section 5.2: Algebra of Linear Transformations

$$T, S: V \rightarrow W$$

$$-(T+S)(v) = T(v) + S(v)$$

$$-T^p v = T \circ T \circ \dots \circ T(p). \quad \rightarrow \text{Multiplication is composition.}$$

$$-D = \frac{d}{dx}$$

↑  
Derivative operator

→ What is the kernel of the operator?

$$D^2 - 2D - 3$$

Approach one:

guess

$$y = e^{\lambda x}$$

$$\Rightarrow \lambda^2 - 2\lambda - 3$$

$$\Rightarrow (\lambda - 3)(\lambda + 1)$$

$$\Rightarrow \lambda = 3, -1$$

$$y = c_1 e^{3x} + c_2 e^{-x}$$

Approach two:

$$D^2 - 2D - 3 = (D - 3)(D + 1) = (D + 1)(D - 3)$$

Find kernel of

$D - 3$  and  $D + 1$

$$y = c_1 e^{3x}, y = c_2 e^{-x}$$

Example!

kernel of  
 $(D - r)^2$ ?

Guess:  $y = x e^{rx}$

$$(D - r)x e^{rx} = e^{rx} + r x e^{rx} - r x e^{rx} \\ = e^{rx}$$

$$(D - r)e^{rx} = 0$$

$$\Rightarrow (D - r)^2 e^{rx} = 0$$

Example:

Find the kernel of:

$$D^4 + D^2 = D^2(D^2 + 1)$$

find kernel of  
 $D^2, D^2 + 1$

$$y = C_1 x + C_2 + C_3 \sin(x) + C_4 \cos(x).$$