# Section 1.6: Properties of Determinants

1. $A$ is invertible $\Leftrightarrow \det(A) \neq 0$.
2. $\det(A \cdot B) = \det(A)\det(B)$.
3. $\det(A^{-1}) = \dfrac{1}{\det(A)}$

~~proof~~:

$$\det(A \cdot A^{-1}) = \det(I)$$
$$\Rightarrow \det(A)\det(A^{-1}) = \det(I)$$
$$\Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}.$$

4. $\det(A^T) = \det(A)$

~~proof~~:

Expanding along row = expanding along column.

**Example:**

For what values of $\lambda$ does the following system have non-trivial solutions

$$x + 3y = \lambda x$$
$$4x + 2y = \lambda y$$

$$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\rightarrow \underbrace{\begin{bmatrix} 1-\lambda & 3 \\ 4 & 2-\lambda \end{bmatrix}}_{A-\lambda I} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det(A - \lambda I) = (1-\lambda)(2-\lambda) - 12$$
$$= \lambda^2 - 3\lambda - 10$$
$$= (\lambda - 5)(\lambda + 2)$$

Therefore, if $\lambda = 5, \lambda = -2$ this system has nontrivial solutions.

Inverses:

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & & \vdots \\ c_{n1} & \cdots & c_{nn} \end{bmatrix}$$ ← cofactor matrix

$$A^{-1} = \frac{1}{\det(A)} C^T.$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow C = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix} \Rightarrow \boxed{\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = A^{-1}}$$