

Math 205  
Quiz #1

1. Consider the following system of equations:

$$\begin{aligned}x + y + 2z &= a, \\x + z &= b, \\2x + y + 3z &= c.\end{aligned}$$

Determine conditions on  $a$ ,  $b$  and  $c$  so that this system of equations has a solution.

The augmented matrix is:

$$\begin{bmatrix} 1 & 1 & 2 & a \\ 1 & 0 & 1 & b \\ 2 & 1 & 3 & c \end{bmatrix} \begin{array}{l} -R1 \\ -2R1 \end{array} \Rightarrow \begin{bmatrix} 1 & 1 & 2 & a \\ 0 & -1 & -1 & b-a \\ 0 & -1 & -1 & c-2a \end{bmatrix} -R2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & a \\ 0 & -1 & -1 & b-a \\ 0 & 0 & 0 & c-b-a \end{bmatrix}$$

Consequently, for solutions to exist,  
 $c = a + b$ .