

Math 205

Quiz #2

1. Let A be the matrix defined by

$$A = \begin{bmatrix} 2 & 0 \\ 4 & a \end{bmatrix},$$

where $a > 0$ is an arbitrary real number. Compute the following

(a) A^2

$$A^2 = \begin{bmatrix} 2 & 0 \\ 4 & a \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 4 & a \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 8+4a & a^2 \end{bmatrix}$$

(b) A^{-1}

$$\left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 4 & a & 0 & 1 \end{array} \right] \xrightarrow{1/2} \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 0 \\ 4 & a & 0 & 1 \end{array} \right] \xrightarrow{-4R_1} \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 0 \\ 0 & a & -2 & 1 \end{array} \right] \xrightarrow{1/a}$$

$$\rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -2/a & 1/a \end{array} \right] \Rightarrow A^{-1} = \begin{bmatrix} 1/2 & 0 \\ -2/a & 1/a \end{bmatrix}.$$

(c) A^{-2}

$$A^{-2} = \begin{bmatrix} 1/2 & 0 \\ -2/a & 1/a \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ -2/a & 1/a \end{bmatrix} = \begin{bmatrix} 1/4 & 0 \\ -1/a + 2/a^2 & 1/a^2 \end{bmatrix}$$

(d) $A^2 - 2I + A^{-2}$

$$A^2 - 2I + A^{-2} = \begin{bmatrix} 4 & 0 \\ 8+4a & a^2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1/4 & 0 \\ -1/a + 2/a^2 & 1/a^2 \end{bmatrix}$$