Math 205
Quiz #5

1. Show that the following sets of $n \times n$ matrices are not subspaces of $M_{n\times n}$.

   (a) The $n \times n$ matrices of determinant zero.

   (b) The $n \times n$ invertible matrices.

(a) Consider

$$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} \text{ and } \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

$\det\left(\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\right)=0$ and $\det\left(\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\right)=0$

but $\det\left(\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\right)=1.$

(b). Consider

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \text{ and } \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$$

which are invertible but

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

which is not invertible.

2. Determine if $\{x^2-1, x^2+1, x^2+x\}$ span the vector space of quadratic polynomials P_2.

Suppose $ax^2+bx+c = c_1(x^2-1)+c_2(x^2+1)+c_3(x^2+x)$

$$= (c_1+c_2+c_3)x^2+c_3x+(c_2-c_1)$$

$$\Rightarrow \begin{bmatrix}1 & 1 & 1\\0 & 0 & 1\\-1 & 1 & 0\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c}1 & 1 & 1 & a\\0 & 0 & 1 & b\\-1 & 1 & 0 & c\end{array}\right] {\scriptstyle +R1} \Rightarrow \left[\begin{array}{ccc|c}1 & 1 & 1 & a\\0 & 0 & 1 & b\\0 & 2 & 1 & c+a\end{array}\right]$$

Therefore, these polynomials span P_2.