

Math 205
Quiz #6

1. The following set of vectors forms a basis for \mathbb{R}^3 :

$$\alpha = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix} \right\}.$$

(a) For the basis α , find $[v]_\alpha$ if

$$v = \begin{bmatrix} 5 \\ -1 \\ 9 \end{bmatrix}.$$

$$\begin{aligned} \begin{bmatrix} 5 \\ -1 \\ 9 \end{bmatrix} &= c_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 9 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & 2 & 3 & | & 5 \\ 2 & 9 & 3 & | & -1 \\ 1 & 0 & 4 & | & 9 \end{bmatrix} &\xrightarrow{\substack{\cdot 2R1 \\ -R1}} \begin{bmatrix} 1 & 2 & 3 & | & 5 \\ 0 & 5 & -3 & | & -11 \\ 0 & -2 & 1 & | & 4 \end{bmatrix} \xrightarrow{+2R2} \begin{bmatrix} 1 & 2 & 3 & | & 5 \\ 0 & 1 & -1 & | & -3 \\ 0 & 0 & -1 & | & -2 \end{bmatrix} \\ \Rightarrow c_3 &= 2 \\ c_2 - 2 &= -3 \Rightarrow c_2 = -1 \\ c_1 + (-2) + 6 &= 5 \Rightarrow c_1 = 1 \end{aligned}$$

$[v]_\alpha = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

(b) Find v , if

$$[v]_\alpha = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}.$$

$$[v]_\alpha = -1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 9 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ 31 \\ 7 \end{bmatrix}.$$