

MST 383/683

Computational Assignment #2

Due Date: October 01, 2021

In this assignment you will use any computer software you would like to solve the following problems. You should print out and submit the output as part of this assignment. The Mathematica notebook entitled Limit Cycle Example on the course website could be useful.

1. The following model has been proposed to model the saturating contact rate:

$$\begin{aligned}\dot{S} &= rS \left(1 - \frac{S}{K}\right) - \frac{\beta IS}{1 + \alpha S} - \mu S \\ \dot{I} &= \frac{\beta IS}{1 + \alpha S} - (\gamma + \mu)I\end{aligned}$$

- (a) Solve for the fixed using a computer algebra system, e.g. Mathematica.
- (b) Using a computer algebra system, calculate the Jacobian for each equilibrium and determine the stability.
- (c) Using a computer algebra system, plot all possible qualitatively different phase portraits that can occur.
- (d) If any of the phase portraits contain a limit cycle, plot at least one solution trajectory on the phase portrait that converges to the limit cycle.
- (e) If any of the phase portraits contain a limit cycle, plot $I(t)$ and $S(t)$ vs t on the same axis for parameter values in which the limit cycle exists.