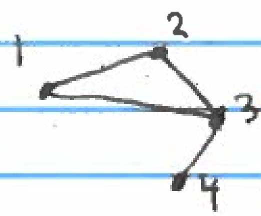
Lecture 8: More Exotic Moment Closures

Clustering:

$$\|M\|_{F_1} = \sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij}$$

A^2 = matrix of triple connections.



$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad A^2 = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

A^2_{ii} = degree of node i

A^2_{ij} = # of triple links connecting node i and j.

$\hookrightarrow A^2_{ij} = \sum_{k=1}^{n} A_{ik}A_{kj}$

$$A^3 = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 3 & 2 & 4 & 1 \\ 4 & 4 & 2 & 3 \\ 1 & 1 & 3 & 0 \end{bmatrix}$$

A^3_{ij} = # of quad connections connecting node i and j.

A^3_{ii} = # of triangles connecting to I.

Clustering coefficient

$$\phi = \frac{\mathrm{Tr}(A^3)}{\|A^2\|_{F_1} - \mathrm{Tr}(A^2)}$$

Exotic Moment Closures

$$[ABC] = \frac{\langle k \rangle - 1}{\langle k \rangle} \frac{[AB][BC]}{[B]} \left((1-\phi) + \phi \frac{n\,[AC]}{\langle k \rangle [A][C]} \right)$$

$$[ABC] = \frac{\langle k \rangle - 1}{\langle k \rangle} \frac{[AB][BC]}{[B]} \left((1-\phi) + \phi \frac{\langle k \rangle [B][C][AC]}{([BB][C] + B[CC])[A][BC} \right.$$