MTH 317/617
Homework #5

Due Date: October 12, 2022

1 Problems for Everyone

3 1. Determine the domain of analyticity for the function

$$f(z) = \mathrm{Log}(z^2 + i)$$

and sketch the branch cuts on the complex plane.

4 2. Find the principal value of the following

(a) $4^{1/2}$

(b) i^{2i}

(c) $(1+i)^{(1+i)}$.

3 3. Find all solutions for $z \in \mathbb{C}$ to the equation

$$\sin(z) = \cos(z).$$

Homework #6

#1.

Determine the domain of analyticity for the function

$$f(z) = Log(z^2 + i)$$

and sketch the branch cuts on the complex plane.

Solution:

f is not analytic when $Re(z^2+i) \le 0$ and $Im(z^2+i) = 0$.

Consequently,

$$x^2 - y^2 \le 0 \quad \text{and} \quad 2xy + 1 = 0$$

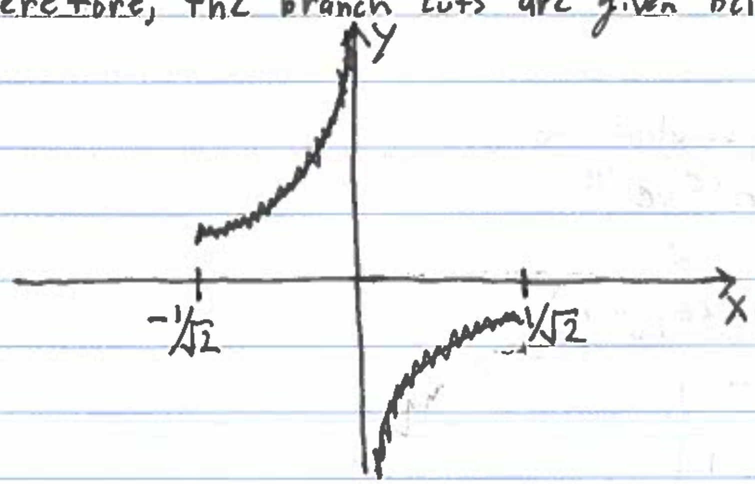
$$\Rightarrow x^2 - y^2 \le 0 \quad \text{and} \quad y = -1/2x$$

$$\Rightarrow x^2 - \frac{1}{4x^2} \le 0 \quad \text{and} \quad y = -1/2x$$

$$\rightarrow 4x^4 \le 1 \quad \text{and} \quad y = -1/2x$$

$$\Rightarrow -1/\sqrt{2} \le x \le 1/\sqrt{2} \quad \text{and} \quad y = -1/2x$$

Therefore, the branch cuts are given below



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#2.

Find the principal value of the following

(a) $4^{1/2}$

(b) i^{2i}

(c) $(1+i)^{1+i}$

Solution:

(a) $4^{1/2} = 2$.

(b) $i^{2i} = e^{2i\,\mathrm{Log}(i)} = e^{2i(\ln(1)+i\pi/2)} = e^{-\pi}$

(c) $(1+i)^{1+i} = e^{(1+i)\,\mathrm{Log}(1+i)} = e^{(1+i)(\ln(\sqrt{2})+i\pi/4)} = e^{\ln(\sqrt{2})-\pi/4}e^{i(\ln(\sqrt{2})+\pi/4)}$.

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#3.

Find all solutions for $z \in \mathbb{C}$ to the equation

$$\sin(z) = \cos(z).$$

Solution:

This equation is equivalent to

$$\frac{e^{iz}-e^{-iz}}{2i} = \frac{e^{iz}+e^{-iz}}{2}$$

$$\Rightarrow e^{iz}-e^{-iz} = i(e^{iz}+e^{-iz})$$

$$\Rightarrow e^{2iz}-1 = ie^{2iz}+i$$

$$\Rightarrow e^{2iz}(1-i) = i+1$$

$$\Rightarrow e^{2iz} = \frac{1+i}{1-i}$$

$$\Rightarrow e^{2iz} = \frac{(1+i)(1+i)}{2} = i = e^{i\pi/2+2n\pi i}$$

$$\Rightarrow z = \pi/4 + n\pi.$$

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