MTH 317/617
Homework #11

Due Date: December 02, 2022

1 Problems for Everyone

1. Verify each of the following.
   (a) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2 e^x}{e^{2x}+1}dx = \frac{\pi^3}{6}$
   (b) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{3e^x + e^{-x}}dx = \frac{\pi}{2\sqrt{3}}$
   (c) $\displaystyle\int_{-\infty}^{\infty} \frac{xe^x}{e^{2x}+1}dx = 0$
2. Verify each of the following
   (a) $\displaystyle\int_{-\infty}^{\infty} \frac{\cos(4x)}{1+x^2}dx = \pi e^{-4}$
   (b) $\displaystyle\int_{-\infty}^{\infty} \frac{x\sin(3x)}{x^2+2}dx = e^{-3\sqrt{2}}\pi$
   (c) $\displaystyle\int_{-\infty}^{\infty} \frac{x\cos(\pi x)}{x^2+x+9}dx = \pi e^{-\frac{\sqrt{35}}{2}\pi}$
3. Verify each of the following
   (a) P.V. $\displaystyle\int_{-\infty}^{\infty} \frac{2x\sin(x)}{x^2-a^2}dx = 2\pi\cos(a),\ a \in \mathbb{R}$
   (b) P.V. $\displaystyle\int_{-\infty}^{\infty} \frac{\sin(ax)}{x-b}dx = \pi\cos(ab),\ a, b \in \mathbb{R}$
   (c) P.V. $\displaystyle\int_{-\infty}^{\infty} \frac{\sin(x)}{x(x^2+1)}dx = \pi\left(1 - \frac{1}{e}\right)$
4. Verify each of the following
   (a) $\displaystyle\int_{0}^{\infty} \frac{\sqrt{x}}{1+x}dx = \pi$
   (b) $\displaystyle\int_{0}^{\infty} \frac{x^\alpha}{x^2-1}dx = \frac{\pi}{2\sin(\pi\alpha)}\left[1 - \cos(\pi\alpha)\right],\ \alpha \in \mathbb{R},\ -1 < \alpha < 1,\ \alpha \neq 0$