

MTH 317/617

Homework #4

Due Date: September 30, 2022

1 Problems for Everyone

1. Show that each of the following functions is nowhere differentiable using definition of the derivative, i.e. do not use the Cauchy Riemann equations.

(a) $f(z) = \operatorname{Re}(z)$

(b) $f(z) = \operatorname{Im}(z)$

(c) $f(z) = |z|$

2. Let $P(z) = (z - z_1)(z - z_2) \cdots (z - z_n)$. Prove that

$$\frac{P'(z)}{P(z)} = \frac{1}{z - z_1} + \cdots + \frac{1}{z - z_n}$$

3. Verify that each given function u is harmonic (in the region where it is defined) and then find the harmonic conjugate of u .

(a) $u = e^x \sin(y)$

(b) $u = xy - x + y$

(c) $u = \sin(x) \cosh(y)$

(d) $u = \operatorname{Im} \exp(z^2)$