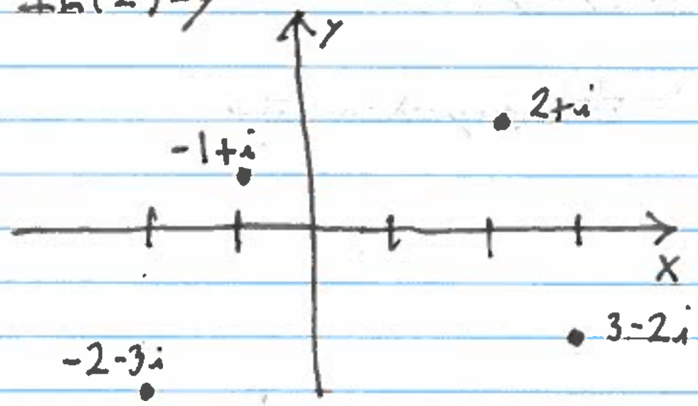
Lecture 2: Point Representation of $\mathbb{C}$.

Cartesian Representation:

$z = x+iy$

$Re(z) = x$

$Im(z) = y$



Sketch

$2+i$

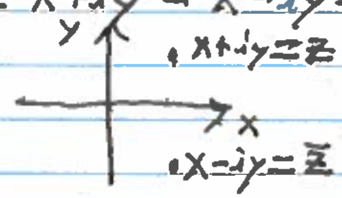
$3-2i$

$-1+i$

$-2-3i$

New operations:

1. $|z| = (x^2+y^2)^{1/2}$ = distance from origin
   * called the modulus or magnitude of z.
2. $\bar{z} = \overline{x+iy} = x-iy$ = reflection about x-axis



Properties:

1. $|z| = (z\cdot\bar{z})^{1/2}$

proof:

$$z\cdot\bar{z} = (x+iy)(x-iy) = x^2+y^2 = |z|^2$$

2. For all $z_1, z_2 \in \mathbb{C}$, $\overline{z_1 z_2} = \bar{z}_1\cdot\bar{z}_2$

proof:

$$\overline{z_1 z_2} = \overline{(x_1+iy_1)(x_2+iy_2)} = \overline{x_1x_2 - y_1y_2 + i(y_1x_2+y_2x_1)}$$

$$= x_1x_2 - y_1y_2 - i(y_1x_2+y_2x_1)$$

$$\bar{z}_1\bar{z}_2 = (x_1-iy_1)(x_2-iy_2) = x_1x_2 - y_1y_2 - i(y_1x_2+y_2x_1)$$

3. $Re(z) = \frac{1}{2}(z + \bar{z})$

proof

$\frac{1}{2}(z + \bar{z}) = \frac{1}{2}(x + iy + x - iy) = x.$ $\blacksquare$

4. $Im(z) = \frac{1}{2i}(z - \bar{z})$

proof:

$\frac{1}{2i}(z - \bar{z}) = \frac{1}{2i}(x + iy - x + iy) = y$ $\blacksquare$