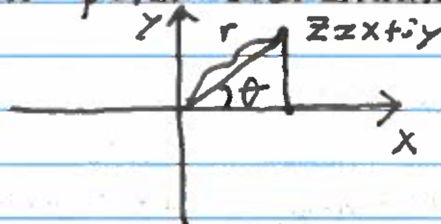
Lecture 3: Vectors and Polar Form

Polar Representation

We can represent

$$z = x + iy$$

in polar coordinates



$x = r\cos\theta$
$y = r\sin\theta$

$r = (x^2+y^2)^{1/2} = |z|$

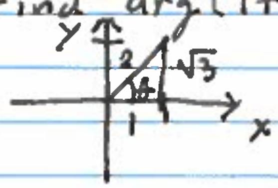
$\theta = \arg(z)$

Compute θ using trig, but be careful!

$$\tan\theta = \frac{y}{x}, \quad \theta \neq \tan^{-1}\left(\frac{y}{x}\right)$$

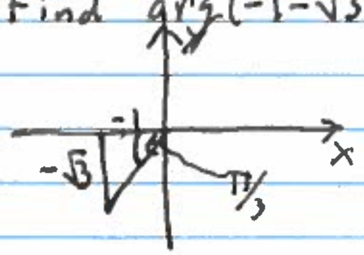
Example:

1. Find $\arg(1+\sqrt{3}i)$.



$\Rightarrow \theta = \pi/3, \ \arg(1+\sqrt{3}i) = \pi/3 + 2n\pi, \ n \in \mathbb{Z}.$

2. Find $\arg(-1-\sqrt{3}i)$



$\Rightarrow \theta = \pi/3 + \pi = 4\pi/3.$

$\Rightarrow \arg(1+\sqrt{3}i) = 4\pi/3 + 2n\pi, \ n \in \mathbb{Z}.$

Summary:

$$\begin{aligned} z &= x + iy \\ &= r\cos\theta + ir\sin\theta \\ &= |z|(\cos(\arg(z)) + i\sin(\arg(z))) \end{aligned}$$

Principal Branch

$\mathrm{Arg}(z)$ selects the unique angle in $(-\pi, \pi]$.

Example:

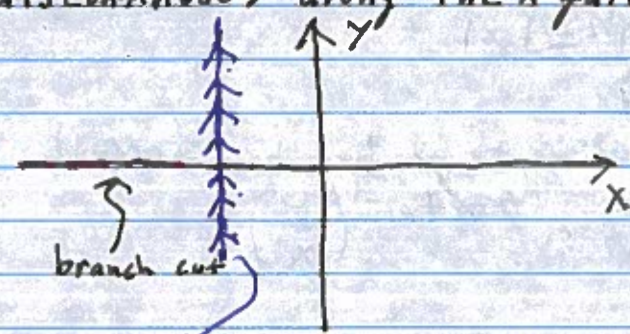
$$\arg(-1-\sqrt{3}i) = 4\pi/3 + 2k\pi,\ k \in \mathbb{Z}$$

$$\mathrm{Arg}(-1-\sqrt{3}i) = 4\pi/3 - 2\pi = -2\pi/3.$$

The function $f: \mathbb{C} \to \mathbb{R}$ defined by

$$f(z) = \mathrm{Arg}(z)$$

is discontinuous along the negative real axis



Imagine we measure $\mathrm{Arg}(z)$ along curve ⇞