

MTH 317/617

Quiz #3

1. Compute the following limit justifying all steps or prove that the limit does not exist.

$$\lim_{z \rightarrow 0} \frac{|z|}{z}$$

Let $z_n = 1/n$ and $w_n = i/n$. Therefore,

$$\frac{|z_n|}{z_n} = \frac{|1/n|}{1/n} = 1 \quad \text{and} \quad \frac{|w_n|}{w_n} = \frac{1/n}{i/n} = -i.$$

Consequently, since $\lim_{n \rightarrow \infty} \frac{|z_n|}{z_n} \neq \lim_{n \rightarrow \infty} \frac{|w_n|}{w_n}$ it follows that $\lim_{z \rightarrow 0} \frac{|z|}{z}$ does not exist.