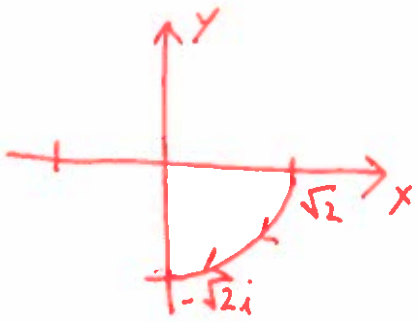
MTH 317/617
Quiz #7

1. Let Γ be the curve in $\mathbb{C}$ parametrized by $z(t) = \sqrt{2}e^{-it/2}$ for $t \in (0, \pi)$.

   (a) Sketch Γ in $\mathbb{C}$.



   (b) Compute the following integral

$$\int_\Gamma e^{2z} dz.$$

$$\int_\Gamma e^{2z} dz = \frac{1}{2} e^{2z} \Big|_{\sqrt{2}}^{-\sqrt{2}i} = \frac{1}{2}\left(e^{-2\sqrt{2}i} - e^{2\sqrt{2}}\right)$$

   (c) Compue the following integral

$$\int_\Gamma \bar{z}^{-1} dz.$$

$$\int_\Gamma \frac{1}{\bar{z}} dz = \int_0^\pi \frac{1}{\sqrt{2}e^{it/2}} \cdot \frac{-i\sqrt{2}}{2} e^{-it/2} dt$$

$$= \frac{-i}{2} \int_0^\pi e^{-it} dt$$

$$= \frac{e^{-it}}{2} \Big|_0^\pi = \frac{e^{-i\pi} - 1}{2} = -1.$$