

## Lecture 12: Love and War

### Example: Romeo and Juliet

$R \sim$  Romeo's love for Juliet

$J \sim$  Juliet's love for Romeo

$$\dot{R} = aJ, \quad a, b > 0$$

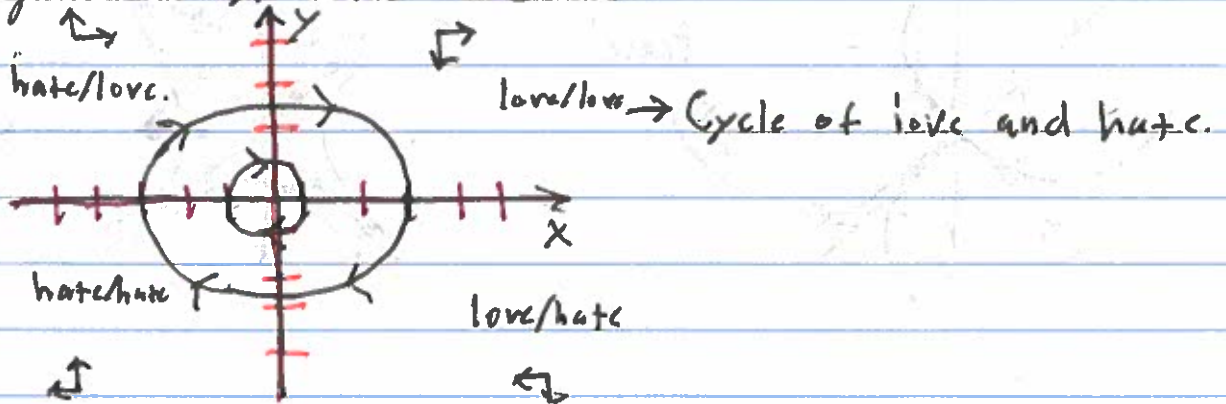
$$\dot{J} = -bR$$

\* Romeo responds positively to Juliet's interest

\* Juliet responds negatively to Romeo's interest

$$A = \begin{bmatrix} 0 & a \\ -b & 0 \end{bmatrix}$$

Eigenvalues  $\lambda = \pm i\sqrt{ab} \Rightarrow$  Center



## Arms Race Between Superpowers

$x \sim$  expenditure of arms by first nation

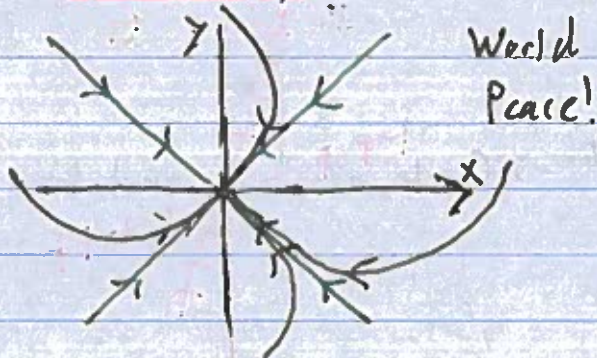
$y \sim$  expenditure of arms by second nation

$\dot{x} = ax + by$ ,  $a < 0$ ,  $\rightarrow$  Reduction of spending on arms towards infrastructure  
 $\dot{y} = bx + ay$ ,  $b > 0$   $\rightarrow$  Expenditure on arms based off of rival

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \Rightarrow \lambda_1 + \lambda_2 = 2a, \Rightarrow \lambda_1 = a + b, \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
$$\lambda_1 \lambda_2 = a^2 - b^2 \quad \lambda_2 = a - b$$

$\lambda_2 < 0$  and we have two cases:

$b < -a \Rightarrow \lambda_1 < 0$



$b > -a \Rightarrow \lambda_1 > 0$

